THE

CANAL

ENGINEER'S AND CONTRACTOR'S

ASSISTANT,

BEING AN EPITOME

OF THE

THEORY OF CANAL CUTTING;

WITH RULES FOR

GAUGING WATER, LOCATING CURVES, CALCULATING TABLES, &c.

ILLUSTRATED BY PROPER EXAMPLES.

TOGETHER WITH A NEW AND EXTENSIVE

Table of Excavation,

ADAPTED TO THE DIMENSIONS

OF THE

PENNSYLVANIA CANAL;

With Examples of its Use, suiting the Comprehension of all Persons employed thereon, and the Manner of applying the same for

CANAL AND TOW-PATH EMBANKMENT.

BY CHARLES POTTS.

Philadelphia:

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1829.

Eastern District of Pennsylvania, to wit:

L. S.

BE IT REMEMBERED, That on the thirtieth day of January, in the fifty-third year of the independence of the United States of America, A. D. 1829, CHARLES POTTS, of the said district, has deposited in this office the title of a book, the right whereof he claims as author, in the words following, to wit:

> The Canal Engineer's and Contractor's Assistant, being an Epitome of the Theory of Canal Cutting; with Rules for gauging Water, locating Curves, calculating Tables, &c., illustrated by proper Examples. Together with a new and extensive Table of Excavation, adapted to the Dimensions of the Pennsylvania Canal; with Examples of its Use, suiting the Comprehension of all Persons employed thereon, and the Manner of applying the same for Canal and Tow-Path Embankment. By Charles Potts.

In conformity to the act of the congress of the United States, entitled "An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned;" and also to the act, entitled "An act supplementary to an act, entitled 'An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned,' and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

D. CALDWELL,
Clerk of the Eastern District of Pennsylvania.

THEORY OF CANAL CUTTING.

ONE of the greatest *desiderata* in cutting canals is, that the stuff, excavated from the bed of the canal, shall, with the least labour and expense, be sufficient to make the adjacent banks.

PROBLEM I.

Given, the profile ABCDEG (fig. 1,) of the parallelogram AG; to find the line S*t*: such that the area, SBC*v*, shall equal *v*DE*t*.

Graphically. Let fall the perpendiculars D*n*, C*m*, and draw the diagonals *mn* and AG; through *p*, the intersection of the diagonals, draw S*t*, parallel to AE or BG, and it is done.

Demonstration. The area *mn*D = CD*n*; hence *mn*GE = CDEG. But (Euclid, lib. 1, prop. 43,) *mn*GE = SBG*t*: Therefore SBG*t* = CDEG, and, taking away the common part C*v*tG we have SBC*v* = *v*DE*t*.

PROBLEM II.

To find the *area* of the *cross section* of a canal, ABCD (fig. 1,) when AD is parallel to the bottom BC; and the banks, or sides, AB, CD slope as $1\frac{1}{2}$ to 1.

Put BC = B; and the depth, or cutting, C*m* = D: Then area ABCD = $D \times \overline{B + \frac{3}{2} D}$. Hence the following

Rule. Add the width of the bottom and base of one slope together, and multiply the sum by the cutting, for the area of the cross section.

Observation. The ratio of $1\frac{1}{2}$ to 1 will be assumed, throughout this work, for the slope of the banks.

Example. What is the area of the cross section of a canal 28 feet bottom, when the cutting is 4 feet?

Here B = 28, D = 4, and $\frac{3}{2}$ D = 6. Therefore $4 \times 28 + 6$ = 136 square feet, the area required.

LEVEL CUTTING.

When the surface of the ground, over which the canal is intended to be carried, is *flat*, or horizontal; the excavation, or digging, is called *level cutting*.

If the excavation, or quantity of earth dug, is equal to the embankment, or quantity of earth necessary to make the adjacent side banks; it is then termed *equal cutting*: and the perpendicular depth, from the surface of the ground to the bottom of the canal, the *depth of equal cutting*.

PROBLEM III.

Given, the profile F*a*ABCDEG (fig. 2,) of a canal, with two banks, and slopes equal: To find the *depth of equal cutting*.

Draw LK, representing the surface of equal cutting: Then, the area *x*BC*y* = areas *x*A*a*L + *y*DEK. Hence the area A*xy*D being added to both, gives ABCD = L*a*EK. Therefore put *a*E = *b* and ABCD = A. Then, TR = $\frac{FG - \sqrt{b^2 + 6A}}{3}$.

Corollary. It is evident that, when the cutting is equal, the length of LK $= \sqrt{b^2 + 6A}$. This line is called the *line of equal cutting*.

Definitions. When the bank, CDEG, is made the *tow-path*, BA*a*F is then denominated the *off-bank*.

The *middle range* (MR) is a line supposed to be drawn on the surface of the ground, always perpendicular to the middle of (BC, fig. 2,) the bottom of the canal.

The lines AB, CD, are termed the canal-bank slopes (CBS) *a*F, the off-bank slope (OBS) and EG, the tow-path slope (TPS).

The horizontal distance, between the slopes *a*F and EG, measured from where the surface of the ground cuts these slopes, as LK, is called the *line of cover* (LC).

When the cutting exceeds the depth of *equal cutting*, as *n*R; the area L*n*K, is denominated the *area of excess*, and is conveniently expressed by the sign +.

When the cutting is less, as *p*R, the area L*p*K, is termed the *area deficient;* and, is expressed by the sign —.

Note. The areas + and — determine the quantity of *spoil bank* that will be made; or can be disposed of, in any portion of the canal.

Example. What is the depth of equal cutting for a canal (Pennsylvania) with 28 feet bottom; tow-path 10 and off-bank 5 feet wide at top, allowing 6½ feet for the height of the banks?

Here we have A $=$ 245.375; $b =$ 62.5 and FG $=$ 82 feet. Whence TR $=$ 2.887 feet, the depth of cutting required. Also LK $=$ 73.34 feet. The area *x*BC*y* $=$ 33.33 square feet.

The following table, showing the horizontal distances, from the MR to the slope lines, for every tenth of a foot cutting, to 6½ feet, is applicable to the dimensions of the Pennsylvania Canal, as given in the above example.

The first column contains the cutting—the second, third, and fourth the corresponding horizontal distances from the middle range to the canal-bank, off-bank, and tow-path slopes: the fifth exhibits the *line of cover*, and the sixth the areas + and — whereby the difference of excavation and embankment, for any section, may be ascertained.

TABLE 1.—LEVEL CUTTING.

CUTTING.	C. B. S.	O. B. S.	T. P. S.	L. C.	AREA + AND —
Feet.	Feet.	Feet.	Feet.	Feet.	Square feet.
.0	14.00	38.50	43.50	82.00	— 224.227
.1	14.15	38.35	43.35	81.70	216.046
.2	14.30	38.20	43.20	81.40	207.887
.3	14.45	38.05	43.05	81.10	199.762
.4	14.60	37.90	42.90	80.80	191.667
.5	14.75	37.75	42.75	80.50	183.606
.6	14.90	37.60	42.60	80.20	175.567
.7	15.05	37.45	42.45	79.90	167.562
.8	15.20	37.30	42.30	79.60	159.587
.9	15.35	37.15	42.15	79.30	151.642
1.0	15.50	37.00	42.00	79.00	143.727
.1	15.65	36.85	41.85	78.70	135.842
.2	15.80	36.70	41.70	78.40	127.987

CUTTING.	C B. S.	O. B. S.	T. P. S.	L. C.	AREA + AND —
Feet.	Feet.	Feet.	Feet.	Feet.	Square feet.
.3	15.95	36.55	41.55	78.10	— 120.162
.4	16.10	36.40	41.40	77.80	112.367
.5	16.25	36.25	41.25	77.50	104.602
.6	16.40	36.10	41.10	77.20	96.867
.7	16.55	35.95	40.95	76.90	89.162
.8	16.70	35.80	40.80	76.60	81.487
.9	16.85	35.65	40.65	76.30	73.842
2.0	17.00	35.50	40.50	76.00	66.227
.1	17.15	35.35	40.35	75.70	58.642
.2	17.30	35.20	40.20	75.40	51.087
.3	17.45	35.05	40.05	75.10	43.562
.4	17.60	34.90	39.90	74.80	36.067
.5	17.75	34.75	39.75	74.50	28.602
.6	17.90	34.60	39.60	74.20	21.167
.7	18.05	34.45	39.45	73.90	13.762
.8	18.20	34.30	39.30	73.60	.6.387
.9	18.35	34.15	39.15	73.30	+ 0.953
3.0	18.50	34.00	39.00	73.00	8.267
.1	18.65	33.85	38.85	72.70	15.553
.2	18.80	33.70	38.70	72.40	22.808
.3	18.95	33.55	38.55	72.10	30.033
.4	19.10	33.40	38.40	71.80	37.228
.5	19.25	33.25	38.25	71.50	44.393
.6	19.40	33.10	38.10	71.20	51.528
.7	19.55	32.95	37.95	70.90	58.633
.8	19.70	32.80	37.80	70.60	65.708
.9	19.85	32.65	37.65	70.30	72.753
4.0	20.00	32.50	37.50	70.00	79.768
.1	20.15	32.35	37.35	69.70	86.753
.2	20.30	32.20	37.20	69.40	93.708
.3	20.45	32.05	37.05	69.10	100.633
.4	20.60	31.90	36.90	68.80	107.528
.5	20.75	31.75	36.75	68.50	114.393
.6	20.90	31.60	36.60	68.20	121.228
.7	21.05	31.45	36.45	67.90	128.033
.8	21.20	31.30	36.30	67.60	134.808
.9	21.35	31.15	36.15	67.30	141.552
5.0	21.50	31.00	36.00	67.00	148.267
.1	21.65	30.85	35.85	66.70	154.952
.2	21.80	30.70	35.70	66.40	161.607
.3	21.95	30.55	35.55	66.10	168.232
.4	22.10	30.40	35.40	65.80	174.827
.5	22.25	30.25	35.25	65.50	181.392
.6	22.40	30.10	35.10	65.20	187.927
.7	22.55	29.95	34.95	64.90	194.432
.8	22.70	29.80	34.80	64.60	200.907
.9	22.85	29.65	34.65	64.30	207.352
6.0	23.00	29.50	34.50	64.00	213.767
.1	23.15	29.35	34.35	63.70	220.152
.2	23.30	29.20	34.20	63.40	226.506
.3	23.45	29.05	34.05	63.10	232.831
.4	23.60	28.90	33.90	62.80	239.126
.5	23.75	28.75	33.75	62.50	245.375

OBLIQUE CUTTING.

When the ground, over which the canal is carried, is sloping, or inclined to the horizon; the excavation is then called *side-long*, or *oblique cutting*.

In oblique cutting there is a point, within the profile of a canal, through which any line being drawn, to represent the surface of the ground, the areas of excavation and embankment will always be equal.

This point is denominated the *centre of cutting*; and its altitude, above the bottom of the canal, the *depth of the centre of cutting*.

PROBLEM IV.

Given the profile of a canal, on side-long ground, with equal slopes;

To find the centre of cutting.

Case 1. When only one bank is made, (fig. 3.)

Graphically. Find by Prob. 1, the line ST, and bisect it in O;

Then O is the centre of cutting.

Demonstration. Let any line HF be drawn through O; then (Prob. 1,) SBCv = vDEF. The triangles OSH, OTF, are similar and equal: Therefore OHBCv = OFEDv; and, taking away the common part Owv, there remains HBCw = wDEF. From D, let fall DN, perpendicular to BG; then OZ will be found by saying, as BG : GN :: DN : OZ — that is,

As total breadth of canal and bank,
Is to width of bank, added to base of one slope,
So is height of bank from bottom,
To the depth of the centre of cutting.

Cor. 1. The depth OZ is constant, whatever may be the slope of HF.

2. The centre of cutting is also the centre of cover; that is, OH is always equal OF.

3. The horizontal distance from O to the middle range, is constant; and Op = ½ (AE — Sv.)

4. The distance Ov is constant, and equal OT — vT.

5. The quantity of excavation above OS is proportional to the altitude Hy: and the quantity below OS is nearly constant; differing from SBCv only by Owv.

6. It is evident that as OSH is always greater than Owv; the area of excavation is, a *minimum*, or the least, in *level cutting*.

Example 1. What is the depth of the centre of cutting for a canal 28 feet bottom: tow-path 10 feet wide, and 6½ feet high? Here AE or BG = 57.5, GN = 19.75, and DN = 6.5. Whence, as 57.5 is to 19.75, so is 6.5 to 2.23, the depth required.

By Cor. Sv = 34.70, OS = OT = 28.75, vT = 22.8, Ov = 5.96 and Op = 11.4 feet.

Forms. For finding the horizontal distances from O to the slopes at HF and w.

For H and F take $\frac{\frac{2}{3}\,ab}{\frac{2}{3}\,b - c}$ For w take $\frac{\frac{2}{3}\,bd}{\frac{2}{3}\,b + c}$

Where a = OS, b = Op, c = pk, and d = Ov.

In particular, if a = 28.75, b = 11.4 and d = 5.96, as in the preceding example, then the above forms become

For H and F $\frac{218.5}{7.6 - c}$ For w $\frac{45.3}{7.6 + c}$.

The facility with which the *sliding rule* may be used, to determine (by the common operation of division) the distances for staking out the canal, must be obvious from the above forms.

For a canal, similar in dimensions to the Pennsylvania Canal, c, is the difference of level in 11.4 feet.—(Ex. 1, Case 1, Prob. iv.)

The following, Table 2, is calculated for such dimensions. The first column gives the value of c, or the fall of the ground in 11.4 feet; the second, the cutting to bottom in the middle range; the third, the horizontal distances from O to F (equal O to H and also half cover); the fourth, the point w in the canal-bank slope; the fifth, the areas of excavation and embankment.

TABLE 2.—EQUAL CUTTING.

VALUE OF C.	CUTTING M. R.	T. P. S.	C. B. S.	AREA, EXCA. AND EMBANK.
Ft. .0	Ft. 2.23	Ft. 28.75	Ft. 5.96	Sq. ft. 69.86
.1	2.33	29.13	5.88	73.29
.2	2.43	29.52	5.80	76.87
.3	2.53	29.93	5.73	80.63
.4	2.63	30.34	5.66	84.52
.5	2.73	30.77	5.59	88.54
.6	2.83	31.21	5.52	90.62
.7	2.93	31.66	5.45	96.80
.8	3.03	32.13	5.39	101.17
.9	3.13	32.61	5.33	105.67
1.0	3.23	33.10	5.27	110.32
1.1	3.33	33.61	5.20	115.09
1.2	3.43	34.13	5.14	119.89
1.3	3.53	34.68	5.09	126.15
1.4	3.63	35.24	5.03	130.52
1.5	3.73	35.81	4.98	135.77
1.6	3.83	36.41	4.92	141.88
1.7	3.93	37.03	4.87	147.42
1.8	4.03	37.67	4.82	153.40
1.9	4.13	38.33	4.77	159.66
2.0	4.23	39.01	4.72	166.21
2.1	4.33	39.72	4.67	172.91
2.2	4.43	40.46	4.62	179.89
2.3	4.53	41.22	4.58	187.19
2 4	4.63	42.00	4.53	194.60
2.5	4.73	42.84	4.48	202.60
2.6	4.83	43.70	4.44	210.76
2.7	4.93	44.59	4.39	219.33
2.8	5.03	45.52	4.35	228.21
2.9	5.13	46.48	4.31	237.37
3.0	5.23	47.50	4.27	247.12
3.1	5.33	48.55	4.23	257.14

Case 2. When there is a *bench*, *berm-bank*, or *towing-path* on the upper side of the canal, as aA, (fig. 4.) then, by adding the bank aAbB to the bank CDEF, the question is reduced to the former case, and its solution effected in the same manner, viz. $s\,o = o\,t$. From d,

let fall dn, perpendicular to bG: Then, for the *depth of the centre of cutting*, in this case say, as aE : Gn :: dn : oz; that is,

As the breadth of canal and banks at top,
Is to the breadth of both banks and base of one slope;
So is the height of the banks,
To the depth of *the centre of cutting*.

Cor. The constant horizontal distance from o, the centre of cutting, to MR, the middle range, in this case, is equal So $-\frac{1}{2}$ Sv.

Note. Several of the corolaries given to this problem in the New Edinburg Encyclopedia, and from thence copied into Sganzin's Civil Engineering, are applicable *only* to a particular case.

Example 1. Given a canal 28 feet bottom; tow-path 10, and berm-bank 5 feet wide, and each $6\frac{1}{2}$ feet high: To find the depth of the centre of cutting.

Here $aE = 62.5$, $Gn = 24.75$ and $dn = 6.5$; Whence, as $62.5 : 24.75 :: 6.5 : 2.574$, the depth required. Also, $so = ot = 31{,}25$, $Sv = 35.72$, $So = 26.25$, $ov = 9.47$ and $op = So - \frac{1}{2} Sv = 8.39$, the distance to middle range.

The general forms given in case 1, serve also for the present case, when the proper values of a, b, c and d, are substituted.

Thus, for a canal as stated in the last preceding example, the forms become, for the horizontal distance from o,

$$\text{To } h \text{ and } f \frac{175}{5.6 - c} \qquad \text{To } w \frac{53}{5.6 + c}$$

c, being in this case, the difference of level for 8.39 feet.

The value of C is very readily obtained, on the ground, with the *levelling instrument.*

Form. To find the *centre of cover*, y (fig. 5,) on any surface, Qf, not passing through o.

$$\frac{\frac{2}{3}b - d}{\frac{2}{3}b - c} \times b \begin{cases} \text{Where } b = op,\ c = pk \text{ and } d = mp, \text{ the cutting in MR less the depth of the centre of cutting.} \\ \text{When } d \text{ is greater than } \frac{2}{3}\ b, \text{ the numerator becomes } d - \frac{2}{3}\ b, \text{ and the point } y \text{ is above } m. \end{cases}$$

PROBLEM V.

Given the profile of a canal with equal slopes, depth of cutting in he middle range, and slope of the ground; to find the horizontal distances from the middle range, at surface, to the slopes.

Let Qf (fig. 5,) represent the surface line, and mR the cutting. Through o, the centre of cutting, draw hF parallel to Qf; produce mR and EF to meet in I; also draw SP parallel to CD: Then say,

For hor. dist. from $\begin{cases} m \text{ to } f, & \text{As } rk : op :: \text{I}m : m\text{N} \\ m \text{ to Q}, & rk : op :: \text{P}p + pm : m\text{L}. \end{cases}$

Example. Let the profile of the canal be as in Ex. 1, Case 2, Prob. iv. Then $op = 8.4$ feet (nearly); let $c = pk$ (the fall of the ground in 8.4 feet) measure the slope of Qf. Put the cutting mR = D, then the preceding analogies become

For the hor. dist. mN	For the hor. dist. mL
$\frac{243.6 - 8.4\ \text{D}}{5.6 - c}$	$\frac{106.4 + 8.4\ \text{D}}{5.6 - c}$

For *deep cutting* the points in the slope line EX will be found, by adding to the first form, 3 times the difference of D and the height of the banks.

The horizontal distances from the middle range of a canal (as given in Ex. 1, Case 2, Prob. iv.) to the *canal bank-slope* CD (fig. 5) will be found by dividing $78.4 + 8.4\ D$ by $5.6 + c$.

Cor. The area $Qf\!hF$ is the *area of excess* or *deficiency*, according as Qf is above or below hF, and varies as og, the perpendicular between them.

PROBLEM VI.

Given the profile of a canal (see Exam. 1, Case 2, Prob. iv.) To find the horizontal distances, when the depth of cutting (D) is increased uniformly the slope of the ground (c) being invariable.

Let the regular increase of cutting be *one-tenth* of a foot. Then for the increase of mL (fig. 5) we have $\frac{106.4 + 8.4\ \overline{D + 1}}{5.6 - c} - \frac{106.4 + 8.4\ D}{5.6 - c}$. Suppose $D = 0$, or the cutting to commence at bottom, then the difference will be .84 divided by $5.6 - c$. The use of this form for constructing tables will be evident from the following example: viz.

Suppose the surface of the ground, or Qf, slopes 5 degrees. Here we shall have $c = .73$ feet: and the foregoing increment becomes equal .172 of a foot. Therefore,

When $D = 0$ we have $m\text{L} = 21.87$ feet,
$D = .1$ we have $m\text{L} = 21.87 + .172 = 22.042$ feet,
$D = .2$ we have $m\text{L} = 22.042 + .172 = 22.214$ feet,

and so on as far as it is thought proper to extend the table.

Cor. It is evident the decrease of mN, for every tenth of a foot increase in cutting, is also equal .172 feet.

Note. The increase of the horizontal distances from the middle range to the slope CD (fig. 5,) for every tenth of a foot cutting will be found by dividing .84 by $5.6 + c$.

PROBLEM VII.

To reduce the section ABCDEF (fig. 6,) of a canal, with equal slopes and a berm-bank, EF, to the equivalent section ABIKF, with one bank only; but wider and shallower; the banking cut down and filled into the bottom.

Put the area DEFG $= A$, GC $= B$, and assume LD $= D$. Then, by question, $D \times \overline{B + \frac{3}{2}\ D} = A$. Therefore, $D = \frac{1}{3}\sqrt{\overline{B^2 + 6A}} - \frac{1}{3}\ B$.

Example. Let the bottom DC $= 28$ feet; berm-bank 5 feet wide and $6\frac{1}{2}$ high. Here $A = 32.5$, $B = 33$ feet; whence $D = .933$ feet, the elevation of the bottom.

Note. The graphical solution given to this problem in the New Edinburg Encyclopedia is correct *only* when the banks slope equally at an angle of 45°, and the base of the bank is equal its height.

The following, Table 3, is calculated for a canal, similar to the Pennsylvania Canal, (Ex. 1, Case 2, Prob. iv.) At the top is given the degrees of slope of the surface (Qf, fig. 5.) The first column contains the cutting in the middle range; the second, third, and fourth, the corresponding horizontal distances therefrom, to the berm-bank (Qb,) tow-path (EF,) and canal-bank (CD,) slopes.

TABLE 3.—OBLIQUE CUTTING.

5 Degrees.				10 Degrees.			
CUT.	B. B. S.	T. P. S.	C. B. S.	CUT.	B. B. S.	T. P. S.	C. B. S.
Ft.	Feet.	Feet.	Feet.	Ft.	Feet.	Feet.	Feet.
.0	21.87	50.12	12.38	.0	25.83	59.12	11.20
.1	22.04	49.94	12.51	.1	26.04	58.91	11.32
.2	22.21	49.77	12.64	.2	26.24	58.71	11.44
.3	22.38	49.60	12.77	.3	26.44	58.50	11.56
.4	22.56	49.43	12.91	.4	26.65	58.30	11.68
.5	22.73	49.26	13.04	.5	26.85	58.10	11.80
.6	22.90	49.08	13.17	.6	27.06	57.89	11.92
.7	23.07	48.91	13.30	.7	27.26	57.69	12.04
.8	23.24	48.72	13.44	.8	27.46	57.48	12.16
.9	23.42	48.57	13.57	.9	27.67	57.28	12.28
1.0	23 59	48.39	13.70	1.0	27.87	57.08	12.40
1.1	23.76	48.22	13.83	1.1	28.08	56.87	12.52
1.2	23.93	48.05	13.97	1.2	28.28	56.67	12.64
1.3	24.11	47.87	*14.10	1.3	28.48	56.46	12.76
1.4	24.28	47.70	14.23	1.4	28.69	56.26	12.88
1.5	24.45	47.53	14.37	1.5	28.89	56.06	13.00
1.6	24.62	47.36	14.50	1.6	29.10	55.85	13.12
1.7	24.80	47.18	14.63	1.7	29.30	55.65	13.24
1.8	24 97	47.01	14.76	1.8	29.50	55.44	13.36
1.9	25.14	46.84	14.90	1.9	29.71	55.24	13.48
2.0	25.31	46.67	15.03	2.0	29.91	55.04	13.60
2.1	25.49	46.49	15.16	2.1	30.12	54.83	13.72
2.2	25.66	46.32	15.29	2.2	30.32	54.63	13.84
2.3	25.83	46.15	15.43	2.3	30.52	54.42	13.96
2.4	26.01	45.98	15.56	2.4	30.73	54.22	*14.08
2.5	26.18	45.80	15.69	2.5	30.93	54.02	14.20
2.6	26.35	45.63	15.83	2.6	31.14	53.81	14.32
2.7	26.52	45.46	15.96	2.7	31.34	53.61	14.44
2.8	26.70	45.29	16.09	2.8	31.54	53.40	14.56
2.9	26.87	45.11	16.22	2.9	31.75	53.20	14.68
3.0	27.04	44.94	16.36	3.0	31.95	53.00	14.80
3.1	27.21	44 77	16.49	3.1	32.16	52.79	14.92
3.2	27.39	44.60	16.62	3.2	32.36	52.59	15.04
3.3	27.56	44.42	16.75	3.3	32 56	52.38	15.16
3.4	27.73	44.25	16.89	3.4	32.77	52.18	15.28
3.5	27.90	44.08	17.02	3 5	32 97	51.98	15.40
3.6	28.08	43.91	17.15	3.6	33.18	51.77	15.52
3.7	28.25	43.73	17.28	3.7	33.38	51.57	15.64
3.8	28.42	43 56	17.42	3.8	33.58	51.36	15.76
3.9	28.59	43.39	17.55	3.9	33.79	51.16	15.88
4.0	28.77	43.22	17.68	4.0	33.99	50.96	16.00
4.1	28.94	43.04	17.82	4.1	34.20	50.75	16.12
4.2	29.11	42.87	17.95	4.2	34.40	50.55	16.24
4.3	29.28	42.70	18.08	4.3	34.60	50.34	16.36
4.4	29.46	42:53	18.21	4.4	34.81	50.14	16.48
4.5	29.63	42.35	18 35	4.5	35.01	49.93	16.60
4 6	29.80	42.18	18.48	4.6	35.22	49.73	16.72
4.7	29.97	42.01	18.61	4.7	35.42	49.53	16.84
4.8	30.15	41.84	18.74	4.8	35.62	49.32	16.96
4.9	30.32	41.66	18.88	4.9	35.83	49.12	17.08
5.0	30.49	41.49	19.01	5.0	36.03	48.92	17.20

ON THE USE OF THE PRECEDING TABLES.

Example 1. The middle range of the canal being located on level ground; to find the distances to *stake off* the cross section, when the cutting is 4 feet.

Enter Table 1 with 4 feet in the first column, and opposite it in the second column will be found 20 feet, *the distances from MR to the slopes of the canal;* in the third column, 32.5 feet, *the distance to off-bank slope;* in the fourth column, 37.5 feet, *the distance to tow-path slope;* in the fifth column, we have 70 feet, the width of ground necessary for the canal and banks; and in the sixth column, the area of excess in square feet. Hence, if 4 feet cutting extends 27 feet, then 79.768 denotes the number of cubic yards of *spoil-bank* that will be made in that distance.

Note. Table 1 may be used for every tenth of a foot cutting, to 13 feet.

Example 2. Let the slope of the ground over which it is intended to cut the canal, be as 11.4 feet to 2 feet.

Enter Table 2 with 2 feet as the value of c; then opposite in column 2 we have 4.23 feet. The middle range therefore being adjusted until the cutting therein becomes equal 4.23 feet, the excavation and embankment will be equal, or in other words, no spoil-bank will be made. To stake off the *cross section*, in column 3, we have 39 feet, the horizontal distance from the centre of cutting to the intersection of the surface and slope of the tow-path; which is also equal the distance to the upper slope of the canal.

In the fourth column we have 4.72 feet, the distance to the lower slope of the canal; and twice the number in column 3, gives 78 feet, the width of ground necessary for the canal and bank.

If the slope and cutting is similar for 27 feet, the numbers (166.21) in column 5, become cubic yards, the excavation in that distance.

Example 3. To stake off the cross section of the canal, when the ground slopes 5 degrees, and the cutting in the middle range is 3 feet.

Enter Table 3 with 3 feet cutting in the first column, and opposite in the second we have 27.04 feet, the horizontal distance from the MR to the intersection of the ground and berm-bank slope.

In column 3 is 44.94 feet to the tow-path slope, and in column 4 we have 16.36 feet to the slopes of the canal.

The width of ground for the canal and banks, when the slope is 5 degrees, is 71.98 feet.

ON EXCAVATION AND EMBANKMENT.

PROBLEM VIII.

To find *the area of the cross section of a canal*, (fig. 7,) put BC $=$ B and MR $=$ D. Then,

Case 1. When the cutting is level. ABCD $=$ D $\times$ $\overline{\text{B} + \frac{3}{2}\text{D}}$, (see Prob. ii.)

Case 2. When the cutting is oblique. Put AM $= t$, and the slope of EH as 1 : n, that is, in n feet (horizontal) let it fall *one* foot. Then AFE $= 6t^2$ divided by $4n^2 - 9$. This added to the area ABCD, will give the area EBCH.

Cor. When $n = 1\frac{1}{2}$, the line EH becomes parallel to EB, and the area AFE becomes infinite or impossible.

Example. What is the area of the cross section of a canal 28 feet bottom, cutting in the middle range 4 feet, when the ground slopes 1 foot in 10?

Here $t = 14 + 6 = 20$ and $n = 10$, therefore AFE $= 2400 \div 391 = 6.11$ square feet. This added to 136 (see Ex. Prob. ii.) gives the area EBCH equal 142.11 square feet.

PROBLEM IX.

To find the *amount of excavation or embankment* in any given length of a canal (fig. 7.)

Case 1. When the ground is level. *Rule:* Multiply the area of the cross section by the given length.

Note. If the area of the cross section be taken in square feet, and the length equal 27 feet, then the number denoting the area, will also be the number of cubic yards in that length.

Case 2. When the ground slopes in the direction or line of the canal. Put the bottom BC $=$ B, length Rr $=$ L, the end cuttings MR $= d$, $mr =$ D and the solidity required $=$ S, then we shall find $S = \frac{1}{2} L (\overline{D + d} \times B + D^2 + Dd + d^2)$. Hence the following

Rule. Multiply the sum of the depths and bottom together, to this add the product and square of each depth, and multiply the whole sum by one half the length.

Cor. If S be required in cubic yards when D, d and B are taken in feet, and L $=$ one chain (66 feet) then $\frac{1}{2} L = \frac{11}{9}$.

Example 1. Required the cubic yards excavation, in one chain length of a canal 28 feet bottom, when the cutting is 6 feet and 4 feet.

Here B $= 28$, D $= 6$, and $d = 4$, whence $S = \frac{11}{9} (\overline{6 + 4} \times 28 + 36 + 24 + 16) = 435\frac{1}{9}$ cubic yards.

Example 2. Required the cubic yards in a piece of *embankment* 1 chain long, 82 feet wide at top, when the ends are 5 and 6 feet high. Here B $= 82$, D $= 6$, and $d = 5$. Therefore $S = \frac{11}{9} (\overline{6 + 5} \times 82 + 36 + 30 + 25) = 1213.666$ cubic yards.

Case 3. When the ground slopes *both* in the direction and across the canal. Let EBCH and *ebch* be the two cross sections, and suppose a plane drawn through A*a* parallel to CD.

Put the prism AFE*afe* $=$ P, the line $am =$ T, and the slope of *eh* as 1 : N, then we shall find the prism

$$P = \frac{1}{3} L \left(\frac{6 T^2}{4 N^2 - 9} + \frac{6 t^2}{4 n^2 - 9} + \sqrt{\frac{6 T^2}{4 N^2 - 9} \times \frac{6 t^2}{4 n^2 - 9}} \right)$$

Rule. Add the areas of each end, and their mean proportional together, and multiply the sum by one-third of the perpendicular length, for the solidity of the prism.

The value of P added to S (Case 2,) will give the solidity of the whole figure.

Example 1. Required the cubic yards in 1 chain of a canal 28 feet bottom, when the middle range cutting is 6 feet and slope 1 in 20 at one end, and 4 feet and slope 1 in 10 feet at the other.

Here AM $= t = 20$ and $n = 10$, $am = T = 23$ and N $= 20$, $\frac{1}{3} L = 22$; therefore $P = 22 \times (1.9 + 6.11 + \sqrt{1.9 \times 6.11}) = 250.8$ cubic feet, or 9.3 cubic yards.

By Ex. 1, Case 2, we have $S = 435\frac{1}{9}$; therefore $S + P = 444.41$ cubic yards, the amount required.

TO LOCATE CIRCULAR ARCS.

When it is found expedient to change the direction of a canal line, ABC, (fig. 8,) from an angular bend, to that of an uniform and regular curve EHF, the following table may be found useful.

TABLE 4.

	45° EF=1.847 HK=.617		50° EF=1.812 HK=.577		55° EF=1.774 HK=.538	
1	.23345	.58970	.22495	.55175	.21644	.51455
2	.45399	50833	.43837	.47617	.42262	.44456
3	.64945	.37774	.62932	.35453	.60876	.33160
4	.80902	.20511	.78801	.19304	.76604	.18104
	60° EF=1.732 HK=.500		65° EF=1.686 HK=.462		70° EF=1.638 HK=.426	
1	.20791	.47815	.19937	.44262	.19081	.40806
2	.40674	.41355	.39073	.38320	.37461	.35360
3	.58779	.30902	.56641	.28683	.54464	.25509
4	.74314	.16913	.71934	.15736	.69466	.14576
	75° EF=1.586 HK=.391		80° EF=1.532 HK=.357		85° EF=1.474 HK=.324	
1	.18224	.37449	.17365	.34202	.16505	.31070
2	.35837	.32482	.34202	.29690	.32557	.26993
3	.52250	.24388	.50000	.22324	.47716	.20313
4	.66913	.13438	.64279	.11335	.61566	.11242
	90° EF=1.414 HK=.292		95° EF=1.350 HK=.262		100° EF=1.284 HK=.234	
1	.15643	.28058	.14781	.25174	.13917	.22423
2	.30902	.24395	.29237	.21902	.27564	.19522
3	.45399	.18390	.43051	.16931	.40674	.14751
4	.58779	.10191	.55919	.09176	.52992	.08201

In the construction of the table, the lines EB and BF are supposed tangents to the curve, ED the radius equal unity, and the arch EHF to be divided into ten equal parts.

At the top of each table is given the angle EBF, chord EF, and HK, the middle ordinate.

In the first column, under each angle, is given the distances KQ, KR, KS, and KT (called abscisses) and in the second, the corresponding perpendiculars to the curve QN, RM, SO, TP, called ordinates.

Lines, cosines, &c. of similar arcs being as the radii of those arcs, the general application of the above table is obvious. Thus, if it be desired to describe a curve with radius 1, foot, yard, chain or the like, then will the abscisses and ordinates be as in the table, indicating feet, yards, chains, or whatever was taken as radius.

If the radius be found 10, 100, 1000, &c. of any measure, then must the decimal points of the numbers given in the table be removed 1, 2, 3, &c. numbers farther back.

The subjoined numbers express the ratio of ED = 1. to the tangents EB or BF for the respective angles of ABC.

Angles	45°	50°	55°	60°	65°	70°
Ratio	.4141	.4663	.5205	.5773	.6371	.7001
Angles	75°	80°	85°	90°	95°	100°
Ratio	.7673	.8390	.9163	1.000	1.091	1.192

The use of these numbers will appear from the following

Example. Let the angle ABC $=$ 60°, and the station EB $=$ 4 chains, then the radius of the curve ED $=$ 4 $\times$ .5773 $=$ 2 chains 30 links.

The abscisses and ordinates therefore for this curve will be found by multiplying the numbers in Table 4, for 60° by 2.3, and the results will be as follows.

The chord EF $=$ 3.98 chain, or 262.68 feet; middle ordinate HK $=$ 1.15 chain, or 75.9 feet; and the

Abscisses.	Feet.	Ordinates.	Feet.
KQ $=$	31.54	QN $=$	72.60
KR $=$	61.64	RM $=$	62.70
KS $=$	89.10	SO $=$	46.86
KT $=$	112.20	TP $=$	25.10

With the aid of a *sliding rule*, these operations of simple multiplication are very expeditiously performed.

ON GAUGING WATER.

Put V $=$ the mean velocity of the stream, measured by the inches it passes over in a second. The *hydraulic mean depth* $=$ R $(= \frac{wh}{b+3.6\,h}$ where w is the mean width of the section, h the mean height or depth, and b the breadth at bottom of a canal,) expressed in inches.

And S $=$ the quotient obtained by dividing the length of the stream, supposing it extended in a straight line, by the difference of the level of its two extremities.

Then the general *formula* for the velocity, V, when uniform, is

$$V = \frac{307\,(\sqrt{R - 0.1})}{\sqrt{S} - \text{hyp. log.}\,\sqrt{S} + 1.6} - 0.3\,\sqrt{R - 0.1}$$

Note. When R and S are both very great, as in large rivers, the decimals under the radical signs, may be neglected without much error.

The *common logarithm* of any number multiplied by 2.302585, will give the hyp. log. of the same number.

If the formula be applied to water running through pipes, then R $=$ $\frac{1}{4}$ of the diameter of the tube.

Example 1. Required the mean velocity of the water in a canal (French creek feeder) 28 feet bottom, 40 top-water surface, and 4 feet deep, when the bottom slopes 3 inches per mile.

Here $w = 34$, $h = 4$, and $b = 28$; whence R $=$ 38.46 inches. Also S $=$ $\frac{1}{3}$ of a mile $=$ 21120 inches, and $\sqrt{S} = 145.327$. The numerator $307\,\sqrt{R - 0.1} = 1901.251$. The negative quantity $0.3\,\sqrt{R - 0.1} = 1.8579$. Again, $\sqrt{S + 1.6} = 145.3327$.

The common log. of 145.3327 $=$ 2.16236, and the hyp. log. $=$ 4.9790. Whence by formula V $=$ 11.68 inches, or nearly one foot per second, the velocity required.

Example 2. The interior diameter of one of the pipes of the water-works near Edinburg, is $4\frac{1}{2}$ inches; length 14930 feet, and head of water (or difference of level of the two extremities) 51 feet. Its maximum discharge was found by experiment to be $11\frac{1}{3}$ cubic feet per minute.

Here R $=$ 1.125 inches, and S $= 292.745 \sqrt{\text{S} + 1.6} = 17.15$. The hyp. log. of 17.15 $= 2.841988 \sqrt{\text{R} - 0.1} = 1.0124$. Hence, by formula V $=$ 21.47 inches per second, or 1288.2 per minute. The area of the pipe $=$ 15.9 square inches, therefore V multiplied by the area of the pipe, and divided by 1728 will give $11\frac{4}{5}$ cubic feet for the discharge per minute, by the formula, which agrees very nearly with the experiment.

Note. The above formula of Du Buat, as rendered in Rees' Cyclopedia, is inaccurate.

The limits of this work will not allow the insertion of several extensive tables calculated for the foregoing formula, whereby the numerator, denominator, and negative quantities, may be obtained from the given values of R and S.

TO FIND THE MEAN VELOCITY OF A STREAM FROM THE SUPERFICIAL.

Rule. Take unity from the square root of the superficial velocity, in the middle of the stream expressed in *inches* per second. The square of the remainder is the *velocity at the bottom;* and half the sum of these two is the *mean velocity*.

Example. Suppose the superficial velocity 25 inches per second. Then $\sqrt{25} - 1 = 4$, the square of which, 16 is the bottom velocity; and half of $25 + 16 = 20\frac{1}{2}$ inches, the mean velocity per second.

THE CANAL PROTRACTOR.

It may be found useful to have a board, of any convenient size, whereon to delineate the profile of the cross-section of the canal, with such other lines, most frequently occurring, in oblique and level cutting.

In the line for the middle range, as IR (fig. 5) should be formed a dove-tailed groove wherein to slide a bar, graduated for feet and decimals, on the same scale as that used for the profile. On the upper end of this bar, should be fixed another bar, as Q*f*, working on *m* as a centre.

On the board should be carefully marked the *centres of equal cutting*, the *line of level cutting*, &c. as determined by Probs. iii. and iv. for the particular dimensions of the canal.

The bar Q*f*, representing the surface line, should be graduated both ways from *m*, the centre of motion. Near the bottom line, R may be formed a *nonius* or *vernier scale*, for taking more accurately the distance *m*R, the cutting in the middle range. A small brass *square*, as *yg*O, should also be made and graduated, the edge *g*O into the scale of feet and decimals, and the edge *yg* as a nonius or vernier for the bar

Qf. This *square* serves for finding the perpendiculars between the centres of cutting and the bar Qf, whereby the areas of excess, Qf, Fh, or of deficiency, are obtained: also, to give the slope to the bar Qf, and to determine more accurately the distances mf and mQ. The manner of using this protractor must be obvious from Prob. v.

The area of the board may be considerably reduced by drawing all the lines on the same side of the groove, IR. Thus contrived, the common *carpenter's rule*, with a pair of compasses for giving the slope to one of the legs, may be made to answer, in a rough manner, the purpose of the protractor.

A set of lines drawn from a point, or the sides of a right angled triangle, drawn and graduated on any spare part of the board, may be used instead of the compasses, to adjust the legs of the rule to the proper slope.

ON THE SIZE AND VELOCITY FOR CANAL BOATS.

From numerous experiments by Borda, Buat and others, it is deduced that the resistance of the water to boats moving at the rate of one foot per second in still water, as that in a canal, is about *one and a half pound* per square foot of the cross section of the boat (immersed in the water) when the ends or prows are *square*, like those of scows, arcs, &c.; and that in the ordinary velocities, the total resistance of *sharp* ended boats is little more than *one-third* of those with square ends, or about half a pound per square foot of the cross section, when the velocity is one foot per second.

Hence, the resistance of fluids to bodies moving therein, being as the square of the velocity; and the maximum action of a horse when his power is employed as in towing, being generally estimated to be $186\frac{2}{3}$ lbs. at the rate of $3\frac{1}{3}$ feet per second.

The area of the cross section of a (*sharp-bowed*) *boat*, to be towed by *one horse*, at the rate of $3\frac{1}{3}$ feet per second, or nearly $2\frac{1}{3}$ miles per hour, will be found by saying,

As $(1)^2 : (3\frac{1}{3})^2 :: \frac{1}{2}$ lb. : $5\frac{5}{9}$ lbs. the resistance per square foot of the cross section of the boat when the action of the horse is a *maximum*. Whence,

Dividing $186\frac{2}{3}$ lbs. by $5\frac{5}{9}$ lbs. we shall have $33\frac{3}{5}$ square feet, the proper area of the cross section of a boat for one horse.

Assuming therefore the draft of a boat to be 3 feet, and the area of that part of the cross section of the boat immersed in the water, a parallelogram, which is nearly the case with boats used on canals.

The proper *width* of such a boat for *one horse* is $33\frac{3}{5}$ divided by 3 feet, that is, $11\frac{1}{5}$ feet.

When the draught is 4 feet, (*Union Canal boats*) the proper width for such a boat is $8\frac{2}{5}$ feet.

The above result is confirmed by the experience of the managers of the Union Canal, see their Report, Nov. 20, 1827.

ON THE SIZE OF CANALS.

The *area* of the cross section of the water in a canal, should not be in a less ratio to the area of the cross section of the boat to be used on it, than 3 to 1, or considerable inconvenience will arise both from the increased resistance to the boat, and the damage to the banks of the canal. (*Chapman's Observations*, p. 102.)

According to M. Du Buat's experiments, when the width of the canal is 4½ times that of the vessel, the resistance is the same as if the boat moved in the open sea. In his experiments, the draught of the boat was about one-fourth less than the depth of water in the canals.

ON EVAPORATION.

The following are the results of several experiments made by Messrs. Hoyle and Dalton, for the purpose of ascertaining the quantity of water evaporated from a surface of earth (covered with grass) and from a like surface of water:

Mean annual rain - - -	33.55 inches.
Mean evaporation from ground -	25.14
Mean evaporation from water -	44.43

From whence it appears, that the evaporation from a surface of water, as in a canal, is nearly twice as much as from green ground, that about 8 inches of rain is left for the supply of springs and rivers; and that the evaporation from a canal or river, exceeds the rain by 10.88 inches.

Fifty cubic feet of water per minute for every mile of canal is usually allowed by the New-York engineers, for evaporation and leakage.

AVERAGE PRICES AS PER CONTRACTS, ESTIMATES, &c. SELECTED FROM THE REPORTS OF THE PENNSYLVANIA CANAL COMMISSIONERS.

	Dolls.	*Cts.*
Excavation, common earth, per cubic yard, - - -		7
Do. hard-pan, do. - - -		16
Do. slate, do. - - -		25
Do. rock, do. - - -		40
Embankment per cubic yard, - - - - -		11
Puddling, do. - - - - -		15
Locks, stone, per foot lift, - - - -	750	00
Do. stone and wood, do. - - - -	396	00
Do. wood, do. - - - - -	150	00
Culverts, mason-work, per perch, - - - -	2	50
Waste-weirs, do. per foot breast, - - -	25	00
Do. wood, do. - - - -	2	75
Waste-gates, stone, per foot opening, - - -	33	00
Rubble-stone work, per perch, - - - -	2	00
Bridges, farm (10 feet wide) wood-work per foot lineal,	2	20
Do. road (20 feet wide) - - - - -	2	75
Dry wall, per perch, - - - - - -		40
Fencing, post and rail, per perch, - - - -		75
Grubbing, per chain, - - - - - -	3	00
Aqueduct at Pittsburg, 1100 feet, wooden trunk, estimated at - - - - - -	100,000	00
Tunnel, Grant's Hill, 800 feet long, 20 feet diameter, per foot lineal, - - - - - -	25	00

DESCRIPTION AND USE OF THE

TABLE

OF EXCAVATION AND EMBANKMENT.

The following, Table 5, shows the number of cubic yards in one chain (66 feet) length of a canal, when the width of the bottom is 28 feet, and the banks or sides slope as $1\frac{1}{2}$ to 1: the depth of cutting at each end of the chain, or the least depth and the difference of the two depths being given.

To use the table. Find the *least* depth of cutting at the *top* of the table, and the *difference* in the *side* column (marked diff.) then under the former, and opposite the latter, will be the number of *cubic* yards, the *excavation.*

If to this *be added* 66 *times the sum of the two heights*, or depths, we have the number of *cubic yards of canal embankment;* or, subtract 22 *times the sum of the two heights, the tow-path embankment*, corresponding to the above canal.

Example 1. How many cubic yards of excavation, when the cutting is 4.5 feet at each end, or the ground level?

Under 4.5 in the *top*, and opposite 0 in the *side* column, is 382.249, the cubic yards of excavation required.

Example 2. Required the cubic yards, when the cutting is 7.2 feet and 11.8 feet; under 7.2, and opposite 4.6 (difference) is 987.604, the cubic yards.

Example 3. How many cubic yards in one chain of *canal embankment*, 82 feet wide at top, 5 feet high at one end, and 6 feet at the other?

Under 5. and opposite 1. in the table, is 487.666

To this add $66 \times 5 + 6 =$ 726

———gives 1213.666, the cubic yards of embankment required.

Example 4. What number of cubic yards in one chain length of *tow-path* embankment, 10 feet wide at top, 5 feet high at one end, and 3 feet at the other?

Under 3. and opposite 2. will be found 333.666

From which take $22 \times 3 + 5 =$ 176.

Leaves the cubic yards of *tow-path* embankment, 157.666.

Note. The third decimal figure given in the table may be repeated at pleasure, if greater accuracy be required.

TABLE 5.—ON EXCAVATION.

Calculated for the Dimensions of the Pennsylvania Canal.

Diff.	.0	.1	.2	.3	.4
.0	0.000	6.881	13.835	20.863	27.964
.1	3.434	10.352	17.343	24.407	31.545
.2	6.893	13.847	20.875	27.976	35.151
.3	10.376	17.367	24.432	31.569	38.781
.4	13.884	20.912	28.013	35.187	42.435
.5	17.416	24.481	31.618	38.829	46.114
.6	20.973	28.074	35.248	42.496	49.817
.7	24.554	31.692	38.903	46.187	53.545
.8	28.159	35.334	42.582	49.903	57.297
.9	31.789	39.001	46.285	53.643	61.074
1.	35.444	42.692	50.013	57.407	64.875
1.1	39.123	46.407	53.765	61.196	68.701
1.2	42.826	50.147	57.542	65.009	72.551
1.3	46.554	53.912	61.343	68.847	76.425
1.4	50.306	57.701	65.168	72.709	80.324
1.5	54.083	61.514	69.018	76.596	84.247
1.6	57.884	65.352	72.893	80.507	88.195
1.7	61.709	69.214	76.792	84.443	92.167
1.8	65.559	73.101	80.715	88.403	96.164
1.9	69.434	77.012	84.663	92.387	100.185
2.	73.333	80.947	88.635	96.396	104.231
2.1	77.256	84.907	92.632	100.429	108.301
2.2	81.204	88.892	96.653	104.487	112.395
2.3	85.176	92.901	100.698	108.569	116.514
2.4	89.173	96.934	104.768	112.676	120.657
2.5	93.194	100.992	108.863	116.807	124.825
2.6	97.239	105.074	112.982	120.963	129.017
2.7	101.309	109.181	117.125	125.143	133.234
2.8	105.404	113.312	121.293	129.347	137.475
2.9	109.523	117.467	125.485	133.576	141.741
3.	113.666	121.647	129.702	137.829	146.031
3.1	117.834	125.852	133.943	142.107	150.345
3.2	122.026	130.081	138.208	146.409	154.784
3.3	126.243	134.334	142.498	150.736	159.147
3.4	130.484	136.612	146.813	155.087	163.535
3.5	134.749	142.914	151.152	159.463	167.947
3.6	139.039	147.241	155.515	163.863	172.384
3.7	143.354	151.592	159.903	168.287	176.845
3.8	147.693	155.967	164.315	172.736	181.331
3.9	152.056	160.367	168.752	177.209	185.841
4.	156.444	164.792	173.213	181.707	190.375
4.1	160.856	169.241	177.698	186.229	194.934
4.2	165.293	173.714	182.208	190.776	199.417
4.3	169.754	178.212	186.743	195.347	204.025
4.4	174.239	182.734	191.302	199.943	208.657
4 5	178.749	187.281	195.885	204.563	213.314
4.6	183.284	191.852	200.493	209.207	217.995
4.7	187.843	196.447	205.125	213.876	222.701
4.8	192.426	201.067	209.782	218.569	227.431
4.9	197.034	205.712	214.463	223.287	232.185
5.	201.666	210.381	219.168	228.029	236.964

Diff.	.5	.6	.7	.8	.9
.0	35.138	42 386	49.707	57.102	64 569
.1	38.756	46.041	53.398	60.829	68.334
.2	42.398	49 719	57.114	64.582	72.123
.3	46.065	53.423	60.854	68.358	75 936
.4	49.756	57 151	64 618	72.159	79 774
.5	53.472	60.903	68 407	75 985	83 636
.6	57 212	64.679	72 221	79 835	87 523
.7	60.976	68.481	76 058	83 709	91.434
.8	64.765	72.306	79.921	87.608	95.369
.9	68 578	76 156	83 807	91 532	99 329
1.	72 416	80.031	87 718	95.479	103.314
1.1	76.278	83.929	91.654	99.452	107.323
1 2	80.165	87.853	95.614	103 448	111 356
1 3	84 076	91.801	99.598	107 469	115 414
1 4	88 012	95.773	103.607	111 515	119.496
1 5	91.972	99 769	107 641	115 585	123 603
1.6	95.956	103 791	111.698	119.679	127 734
1.7	99 965	107.836	115.781	123.798	131.889
1.8	103 998	111.906	119.887	127.942	136.069
1.9	108.056	116 001	124.018	132 109	140 274
2.	112.138	120.119	128.174	136.302	144.503
2.1	116.245	124.263	132 354	140.518	148.756
2.2	120.376	128.431	136.558	144 759	153 034
2.3	124.532	132 623	140.787	149 025	157 336
2.4	128 712	136 839	145.041	153.315	161.663
2.5	132.916	141.081	149.318	157.629	166.014
2.6	137.145	145 346	153 621	161.968	170.389
2.7	141.398	149.636	157.947	166.332	174 789
2.8	145.678	153 951	162.298	170 719	179.214
2.9	149 978	158.289	166.674	175.432	183 663
3.	154 305	162.653	171.074	179.568	188.136
3.1	158 656	167.041	175 498	184.029	192 634
3.2	163 032	171.453	179.947	188.415	197.156
3.4	167.432	175.889	184 421	193 025	201.703
3.4	171.856	180.351	188 918	197 559	206.274
3.5	176.305	184 836	193.441	202.118	210 869
3 6	180.778	189.346	197.987	206.702	215 489
3 7	185 276	193 881	202.558	211.309	220.134
3 8	189 798	198.439	207.154	215 942	224 803
3.9	194.345	203.023	211.774	220.598	229.496
4.	198.916	207.631	216 418	225 279	234.214
4 1	203.512	212.263	221 087	229.985	238 956
4.2	208 132	216.919	225 781	234 715	243 723
4 3	212 776	221 601	230.498	239 469	248.514
4 4	217 445	226.306	235.241	244.248	253.329
4.5	222 138	231.036	240.007	249.052	258.169
4 6	226.856	235.791	244.798	253.879	263.034
4 7	231.598	240 569	249 614	258.732	267.923
4 8	236.365	245.373	254.454	263 608	272.836
4 9	241.156	250.201	259.318	268.509	277.774
5.	245 972	255 053	264.207	273 435	282.736

Diff.	1.0	1.1	1.2	1.3	1.4
.0	72.111	79.725	87.413	95.174	103.008
.1	75.912	83.563	91 287	99.085	106.956
.2	79.737	87.425	95 186	103.021	110.928
3	83 587	91.312	99.109	106 981	114.925
.4	87.462	95.223	103.057	110.965	118.946
.5	91.361	99 158	107.029	114.974	122.992
.6	95.284	103 118	111 026	119.007	127.062
.7	99.232	107.103	115.047	123 065	131.156
.8	103.204	111.112	119.093	127 147	135.275
.9	107 201	115.145	123.163	131.254	139.418
1.	111 222	119.203	127 257	135.385	143 586
1.1	115.267	123.285	131.376	139.541	147 778
1.2	119.337	127.392	135.519	143 721	151.995
1 3	123 432	131 523	139.687	147.925	156.236
1 4	127.551	135.678	143.879	152.154	160.502
1.5	131.694	139.858	148 096	156 407	164.792
1.6	135.862	144.063	152.337	160.685	169.106
1 7	140 054	148.292	156 603	164.987	173.445
1 8	144.271	152.545	160.893	169.314	177 808
1 9	148 512	156 823	165.207	173.665	182.196
2.	152.777	161.125	169.546	178 041	186.608
2.1	157.067	165 452	173 909	182.441	191.045
2.2	161 382	169 803	178.297	186 865	195 506
2 3	165.721	174.178	182.709	191.314	199.992
2.4	170 084	178.578	187.146	195 787	204 502
2.5	174.472	183 003	191.607	200 285	209 036
2 6	178.884	187.452	196 093	204 807	213.595
2.7	183 321	191.925	200.603	209.354	218.178
2.8	187.782	196 423	205 137	213.925	222.786
2 9	192 267	200 945	209.696	218 521	227 418
3.	196 777	205 492	214 279	223 141	232.075
3.1	201.312	210 063	218 887	227.785	236 756
3 2	205.871	214 658	223 519	232.454	241.462
3 3	210.454	219 278	228 176	237.147	246.192
3.4	215.062	223 923	232.857	241.865	250.946
3.5	219.694	228.592	237 563	246 607	255.725
3 6	224 351	233.285	242 293	251.374	260.528
3 7	229.032	238.003	247 047	256.165	265.356
3 8	233 737	242.745	251.826	260.981	270 208
3.9	238.467	247 512	256 629	265 821	275.085
4.	243 222	252.303	261 457	270.685	279.986
4 1	248 001	257.118	266.309	275.574	284 912
4 2	252 804	261.958	271 186	280.487	289.862
4 3	257.632	266 823	276.087	285.425	294.836
4 4	262 484	271 712	281.013	290 387	299.835
4.5	267.361	276 625	285.963	295.374	304 858
4.6	272 262	281 563	290.937	300 585	309.906
4.7	277.187	286 525	295 936	305.421	314.978
4.8	282.137	291 512	300.959	310.481	320.075
4 9	287 112	296.523	306 007	315 565	325.196
5.	292.111	301.558	311 079	320.674	330 342

Diff.	1.5	1.6	1.7	1.8	1.9
.0	110.916	118.897	126.952	135.079	143.281
.1	114.901	122.918	131.009	139.174	147 412
.2	118.909	126.964	135 092	143.293	151.567
.3	122.943	131.034	139.198	147.436	155.747
.4	127.001	135.128	143.329	151.604	159 952
.5	131.083	139.247	147.485	155.796	164.181
.6	135.189	143 391	151.665	160.013	168.434
.7	139.321	147 558	155.869	164.254	172.712
.8	143.476	151 751	160.098	168.519	177.014
.9	147.656	155.967	164.352	172 809	181.341
1.	151.861	160.208	168.629	177.124	185 692
1.1	156.089	164.474	172 932	181.463	190 067
1.2	160.343	168.764	177.258	185 826	194.467
1 3	164.621	173.078	181.609	190.214	198.892
1.4	168.923	177.417	185 985	194.626	203.341
1.5	173.249	181.781	190 385	199.063	207.814
1.6	177.601	186 168	194.809	203.524	212.312
1.7	181.976	190 581	199.258	208.009	216.834
1.8	186.376	195 017	203.732	212.519	221.381
1.9	190.801	199.478	208.229	217.054	225.952
2.	195.249	203.964	212.752	221.613	230.547
2.1	199.723	208.474	217.298	226.196	235.167
2.2	204 221	213.008	221.869	230.804	239.812
2.3	208.743	217.567	226.465	235.436	244.481
2.4	213.289	222.151	231.085	240 093	249.174
2.5	217 861	226.758	235 729	244 774	253.892
2 6	222 456	231.391	240.398	249.479	258.634
2.7	227.076	236.047	245.092	254.209	263.401
2.8	231.721	240.728	249 809	258 964	268.192
2.9	236 389	245 434	254.552	263.743	273.007
3.	241.083	250.164	259.318	268 546	277.847
3 1	245.801	254 918	264.109	273.374	282.712
3 2	250 543	259 697	268.925	278.226	287.601
3.3	255.309	264.501	273.765	283 103	292.514
3.4	260 101	269 328	278.629	288.004	297 452
3 5	264 916	274.181	283.518	292.929	302.414
3.6	269 756	279 057	288 432	297.879	307 401
3.7	274.621	283.988	293.369	302 854	312.412
3 8	279.509	288.884	298 332	307.853	317.447
3.9	284.423	293 834	303 318	312.876	322.507
4.	289 361	298 808	308.329	317.924	327.592
4 1	294.323	303 807	313.365	322.996	332.701
4.2	299.309	308.831	818.425	328.093	337.834
4 3	304.321	313.878	323.509	333.214	342 992
4 4	309 356	318 951	328 618	338 359	348.174
4 5	314.416	324.047	333 752	343.529	353.381
4 6	319.501	329.168	338 909	348.724	358.612
4 7	324 609	334.314	344 092	353 943	363.867
4.8	329.743	339.484	349 298	359.186	369 147
4.9	334 901	344.678	354 529	364 454	374 452
5.	340.083	349.897	359.785	369 746	379.781

Diff.	2.0	2.1	2.2	2.3	2.4
.0	151.555	159.903	168.324	176.818	185.386
.1	155.723	164 107	172 565	181.096	189.701
.2	159.915	168.336	176.831	185.398	194.039
.3	164.132	172 589	181.121	189 725	198.403
.4	168.373	176.867	185.435	194.076	202.791
.5	172 638	181.169	189.774	198.452	207.203
.6	176.928	185 496	194.137	202.852	211.639
.7	181.243	189.847	198.525	207.276	216.101
.8	185.582	194 223	202.937	211.725	220 586
.9	189.945	198.623	207.374	216.198	225.096
1.	194 333	203.047	211.835	220.696	229.631
1.1	198 745	207.496	216.321	225.218	234.189
1 2	203.182	211.969	220 831	229.765	238.773
1.3	207.643	216.467	225.365	234.336	243.381
1.4	212.128	220.989	229.924	238.932	248.013
1.5	216.638	225.536	234.507	243.552	252.669
1.6	221.173	230 107	239.115	248.196	257.351
1.7	225.732	234.703	243 747	252 865	262.056
1.8	230.315	239.323	248.404	257.558	266.786
1.9	234.923	243.967	253 085	262.276	271.541
2.	239.555	248 636	257.791	267.018	276.319
2.1	244.212	253.329	262.521	271.785	281.123
2.2	248.893	258.047	267.275	276.576	285.951
2.3	253.598	262.789	272.054	281.392	290.803
2.4	258.328	267.556	276.857	286.232	295.679
2.5	263.083	272.347	281.685	291.096	300.581
2.6	267.862	277.163	286.537	295.985	305.506
2.7	272.665	282.003	291.414	300.898	310.456
2.8	277.493	286.867	296.315	305 836	315.431
2.9	282.345	291.756	301.241	310.798	320.429
3.	287.222	296.669	306.191	315.785	325.453
3.1	292.123	301.607	311.165	320.796	330.501
3.2	297 048	306.569	316.164	325.832	335.573
3.3	301.998	311.556	321.187	330.892	340.669
3.4	306.973	316.567	326.235	335.976	345.791
3.5	311.972	321.603	331.307	341.085	350.936
3.6	316 995	326.663	336.404	346.218	356.106
3.7	322.043	331.747	341.525	351.376	361 301
3.8	327.115	336.856	346 671	356 558	366.519
3.9	332.212	341.989	351.841	361.765	371.763
4.	337.333	347.147	357.035	366.996	377.031
4.1	342.478	352.329	362.254	372 252	382.323
4.2	347.648	357.536	367.497	377.532	387.639
4.3	352 843	362.767	372.765	382.836	392.981
4.4	358.062	368.023	378.057	388.165	398.346
4.5	363.305	373.303	383 374	393.518	403.736
4.6	368.573	378.607	388.715	398.896	409.151
4 7	373 865	383.936	394.081	404.298	414.589
4.8	379.182	389 289	399.471	409.725	420.053
4.9	384.523	394.667	404 885	415.176	425.541
5.	389.888	400.069	410.324	420.652	431.053

Diff.	2.5	2.6	2.7	2.8	2.9
.0	194.027	202.742	211.529	220 391	229.325
.1	198.378	207.129	215.954	224.852	233.823
.2	202.754	211.542	220.403	229.337	238.345
.3	207.154	215.978	224.876	233 847	242 892
.4	211.578	220.439	229.374	238.382	247.463
.5	216 027	224.925	233.896	242.941	252 058
.6	220.501	229.435	238.443	247.524	256.678
.7	224.998	233.969	243.014	252 132	261.323
.8	229 521	238.528	247 609	256.764	265 992
.9	234.067	243 112	252.229	261.421	270.685
1.	238.638	247 719	256.874	266.102	275.403
1.1	243.234	252.352	261 543	270.807	280.145
1.2	247.854	257.008	266.236	275 537	284.912
1.3	252.498	261.689	270.954	280.292	289.703
1.4	257.167	266.395	275 696	285.071	294 518
1.5	261.861	271.125	280 463	289 874	299 358
1.6	266.578	275 879	285.254	294 702	304.223
1.7	271.321	280 658	290.069	299.554	309.112
1.8	276.087	285.462	294.909	304.431	314 025
1.9	280 878	290.289	299.774	309.332	318.963
2.	285 694	295.142	304.663	314.257	323.925
2 1	290.534	300 018	309.576	319.207	328.912
2.2	295 398	304 919	314.514	324.182	333.923
2 3	300.287	309 845	319.476	329.181	338 958
2 4	305.201	314 795	324 463	334 204	344 018
2.5	310.138	319.769	329.474	339.252	349.103
2.6	315.101	324 763	334.509	344.324	354 212
2.7	320.087	329.792	339.569	349.421	359.345
2.8	325.098	334.839	344.654	354.542	364.503
2.9	330.134	339.912	349.763	359 687	369.685
3.	335.194	345.008	354.896	364 857	374.892
3.1	340.278	350.129	360.054	370.052	380.123
3.2	345.387	355.275	365.236	375.271	385.378
3 3	350.521	360.445	370.443	380.514	390.658
3.4	355.678	365.639	375.674	385.782	395.963
3.5	360.861	370.858	380.929	391.074	401 292
3.6	366.067	376.102	386 209	396.391	406.645
3 7	371.298	381.369	391.514	401.732	412.023
3.8	376 554	386 662	396.843	407.097	417.425
3.9	381.834	391.978	402.196	412.487	422.852
4.	387.138	397.319	407 574	417 902	428.303
4.1	392.467	402.685	412 976	423.341	433.778
4.2	397.821	408.075	418 403	428 804	439 278
4.3	403.198	413 489	423.854	434 292	444 803
4.4	408.601	418.928	429.329	439 804	450.352
4 5	414.027	424 392	434.829	445.341	455.925
4.6	419.478	429.879	440 354	450.902	461.523
4.7	424.954	435.393	445 903	456 487	467 145
4.8	430.454	440.928	451.476	462.097	472.792
4.9	435 978	446.489	457.074	467.732	478.463
5.	441.527	452.075	462.696	473 391	484 158

Diff.	3.0	3.1	3.2	3.3	3.4
.0	238.333	247.414	256.568	265.796	275.097
.1	242 867	251.985	261.176	270.441	279.778
.2	247.426	256.581	265.808	275 109	284.484
.3	252 009	261.201	270.465	279 803	289.214
.4	256 617	265.845	275 146	284.521	293 968
.5	261.249	270.514	279.852	289.263	298.747
.6	265.906	275.207	284.582	294.029	303.551
.7	270.587	279 925	289.336	298 821	308 378
.8	275.293	284.667	294.115	303.636	313.231
.9	280 023	289.434	298 918	308.476	318.107
1.	284 777	294.225	303.746	313 341	323.008
1.1	289.556	299.041	308 598	318.229	327.934
1.2	294.359	303.881	313.475	323.143	332.884
1.3	299.187	308 745	318.376	328.081	337.858
1.4	304.039	313.634	323.302	333.043	342.857
1.5	308.916	318.547	328.252	338.029	347.881
1.6	313.817	323.485	333.226	343.041	352.928
1.7	318.743	328.447	338.225	348 076	358.001
1.8	323.693	333.434	343.248	353.136	363.097
1.9	328.667	338.445	348.296	358.221	368.218
2.	333.666	343.481	353.368	363.329	373.364
2.1	338 689	348.541	358.465	368.463	378 534
2.2	343.737	353.625	363.586	373.621	383.728
2 3	348.809	358.734	368 732	378.803	388 947
2.4	353.906	363 867	373.902	384.009	394.191
2.5	359.027	369.025	379.096	389.241	399.458
2.6	364.173	374 207	384 315	394.496	404.751
2.7	369 343	379.414	389 558	399.776	410.067
2.8	374.537	384 645	394.826	405 081	415.408
2 9	379 756	389.901	400.118	410.409	420.774
3.	384.999	395.181	405.435	415.763	426.164
3.1	390.267	400 485	410.776	421.141	431.578
3 2	395.559	405.814	416 142	426.543	437.017
3.3	400.876	411.167	421.532	431.969	442.481
3.4	406.217	416.545	426.946	437.421	447.968
3.5	411.583	421.947	432 385	442.896	453.481
3.6	416.973	427 374	437.848	448 396	459 017
3 7	422.387	432.825	443.336	453.921	464.578
3 8	427 826	438.301	448 848	459.469	470.164
3.9	433.289	443.801	454 385	465.043	475.774
4.	438.777	449.325	459.946	470.641	481.408
4.1	444.289	455.874	466.532	476.263	487.067
4.2	449.826	460.447	471.142	481.909	492.751
4.3	455 387	466 045	476.776	487.581	498.458
4.4	460.973	471.667	482.435	493.276	504.191
4.5	466 583	477.314	488.118	498.996	509.947
4.6	472 317	482.985	493.826	504.741	515 728
4.7	477 876	488 681	499 558	510.509	521.534
4 8	483.559	494.401	505 315	516.303	527.364
4 9	489.267	500.145	511 096	522.121	533.218
5.	494.999	505.914	516 902	527.963	539 097

Diff.	3.5	3.6	3.7	3.8	3.9
.0	284.472	293.919	303.441	313.035	322.703
.1	289.189	298.674	308.232	317.863	327.567
.2	293.932	303 453	313 047	322.715	332.456
.3	298.698	308 256	317.887	327.592	337.369
.4	303.489	313.084	322 752	332.493	342.307
.5	308.305	317.936	327 641	337.418	347.269
.6	313.145	322.813	332.554	342.368	352.256
.7	318.009	327.714	337 492	347.343	357.267
.8	322.898	332.639	342.454	353.342	362.303
.9	327.812	337.589	347.441	357.365	367.363
1.	332 749	342.564	352 452	362.413	372.447
1.1	337.712	347.563	357.487	367.485	377.556
1.2	342.698	352 586	362 547	372 582	382.689
1.3	347.709	357.634	367.632	377.703	387.847
1.4	352 745	362.706	372.741	382.848	393.029
1.5	357.805	367.803	377.874	388 018	398.236
1 6	362.889	372.924	383.032	393 213	403.467
1 7	367.998	378.069	388.214	398.432	408 723
1.8	373.132	383.239	393.421	403.675	414.003
1.9	378 289	388.464	398.652	408.943	419.307
2.	383.472	393.653	403.907	414.235	424 636
2.1	388.678	398.896	409.187	419.552	429 989
2.2	393.909	404 164	414.492	424 893	435.367
2.3	399.165	409.456	419 821	430.258	440.769
2.4	404.445	414.773	425.174	435 648	446.196
2.5	409.749	420.114	430 552	441.063	451.647
2.6	415.078	425.479	435 954	446 502	457.123
2 7	420.432	430.869	441.381	451.965	462.623
2 8	425.809	436.284	446.832	457.453	468.147
2.9	431.212	441.723	452.307	462.965	473.696
3.	436.638	447.186	457.807	468.502	479.269
3.1	442.089	452 674	463.332	474.063	484.867
3.2	447.565	458.186	468.881	479.648	490.489
3.3	453.065	463.723	474.454	485.258	496.136
3.4	458.589	469.284	480.052	490.893	501.807
3 5	464.138	474.869	485 674	496 552	507.503
3.6	469.712	480.479	491.321	502.235	513.223
3.7	475.309	486.114	496.992	507.943	518.967
3.8	480.932	491.773	502.687	513.675	524.736
3 9	486 578	497.456	508.407	519.432	530.529
4.	492.249	503.164	514 152	525.213	536.347
4.1	497.945	508 896	519 921	531 018	542.189
4.2	503.665	514.653	525 714	536.848	548.056
4.3	509.409	520.434	531.532	542.703	553 947
4.4	515.178	526.239	537 374	548.582	559·863
4 5	520 972	532.069	543.241	554.485	565.803
4 6	526 789	537 924	549.132	560.413	571.767
4 7	532.632	543.803	555.047	566.365	577.756
4.8	538.498	549.706	560.987	572.342	583.769
4.9	544.389	555.634	566.952	578.343	589.807
5.	550.305	561.586	572.941	584 368	595.869

Diff.	4.0	4.1	4.2	4.3	4.4
.0	332.444	342.258	352.146	362.107	372.142
.1	337.345	347.196	357.121	367.118	377.189
.2	342.271	352.158	362.119	372.154	382.262
.3	347.221	357.145	367.143	377.214	387.358
.4	352.195	362.156	372.191	382.298	392.479
.5	357.194	367.192	377.263	387 407	397.625
.6	362.217	372 252	382.359	392.541	402.795
.7	367.265	377.336	387.481	397.698	407.989
.8	372 337	382.445	392.626	402.881	413.208
.9	377.434	387.578	397.796	408.087	418.452
1.	382 555	392.736	402.991	413 318	423.719
1.1	387.701	397.918	408.209	418.574	429.012
1 2	392.871	403 125	413.453	423.854	434 328
1.3	398.065	408 356	418.721	429.158	439.669
1.4	403 284	413 612	424.013	434.487	445.035
1.5	408.527	418.892	429.329	439.841	450 425
1.6	413.795	424.196	434.671	445 218	455.839
1.7	419.087	429 525	440.036	450.621	461.278
1.8	424.404	434.878	445.426	456.047	466 742
1.9	429.745	440.256	450.841	461.498	472 229
2.	435.111	445.658	456.279	466.974	477.742
2.1	440.501	451.085	461.743	472.474	483.278
2.2	445.915	456 536	467.231	477.998	488.839
2.3	451.354	462.012	472.743	483.547	494.425
2.4	456 817	467.512	478.279	489.121	500.035
2.5	462.305	473.036	483.841	494.718	505.669
2.6	467.817	478.585	489.426	500.341	511.328
2.7	473.354	484 158	495.036	505.987	517.012
2.8	478.915	489 756	500.671	511.658	522.719
2.9	484.501	495 378	506.329	517.354	528.452
3.	490.111	501 025	512.013	523.074	534.208
3.1	495.745	506.696	517.721	528.818	539.989
3.2	501.404	512.392	523.453	534.587	545.795
3.3	507.087	518 112	521.209	540.381	551.625
3.4	512.795	523.856	534.991	546.198	557.479
3.5	518.527	529.625	540.796	552.041	563.358
3 6	524.284	535 418	546.626	557.907	569 262
3 7	530.065	541.236	552.481	563 798	575.189
3.8	535.871	547 078	558.359	569.714	581.142
3.9	541.701	552.945	564.263	575.654	587.118
4.	547.555	558.836	570.191	581.618	593.119
4.1	553.434	564.752	576.143	587.607	599.145
4.2	559.337	570.692	582.119	593.621	605 195
4.3	565.265	576.656	588.121	599.658	611.269
4.4	571.217	582 645	594 146	605.721	617.368
4.5	577.194	588.658	600.196	611.807	623.492
4 6	583.195	594.696	606 271	617.918	629.639
4.7	589.221	600 758	612.369	624.054	635.812
4.8	595.271	606.845	618.493	630.214	642.008
4.9	601.345	612.956	624.641	636.398	648.229
5.	607.444	619.092	630.813	642.607	654.475

Diff.	4.5	4.6	4.7	4.8	4.9
.0	382.249	392 431	402.685	413.013	423.414
.1	387.334	397.552	407.843	418.207	428 645
.2	392.443	402 697	413.025	423 426	433 901
.3	397.576	407.867	418.232	428.669	439.181
.4	402.734	413.062	423 463	433 937	444 485
.5	407.916	418.281	428.718	439 229	449.814
.6	413.123	423.524	433.998	444.546	455 167
.7	418 354	428.792	439.303	449.887	460 545
.8	423.609	434.084	444.632	455 253	465.947
.9	428.889	439.401	449.985	460.643	471.374
1.	434.194	444.742	455.363	466.057	476 825
1.1	439.523	450.107	460.765	471.496	482.301
1 2	444.876	455.497	466.192	476.959	487.801
1 3	450.254	460.912	471.643	482.447	493.325
1.4	455.656	466.351	477.118	487.959	498.874
1.5	461.083	471.814	482.618	493.496	504.447
1.6	466 534	477.302	488.143	499.057	510.045
1.7	472.009	482.814	493.692	504.643	515.667
1.8	477 509	488.351	499.265	510.253	521.314
1.9	483 034	493.912	504.863	515 887	526.985
2.	488.583	499.497	510.485	521.546	532 681
2.1	494.156	505.107	516.132	527.229	538 401
2.2	499.754	510.742	521.803	532 937	544.145
2.3	505.376	516.401	527.498	538.669	549.914
2.4	511.023	522.084	533.218	544.426	555 707
2.5	516.694	527.792	538 963	550.207	561 525
2.6	522.389	535.524	544.732	556.013	567.367
2.7	528.109	539.281	550.525	561.843	573.234
2.8	533.854	545.062	556.343	567.697	579.125
2.9	539 623	550.867	562.185	573.576	585.041
3.	545.416	556.697	568.052	579.479	590.981
3.1	551.234	562.552	573.943	585.407	595.945
3.2	557.076	568.431	579.858	591.359	602 934
3.3	562.943	574.334	585.798	597.336	608 947
3.4	568.834	580.262	591.763	603 337	614.985
3.5	574 749	586 214	597.752	609.363	621.047
3.6	580.689	592.191	603 765	615.413	627.134
3.7	586.654	598.192	609.803	621.487	633 245
3.8	592.643	604 217	615.865	627.586	639.381
3.9	598.656	610 267	621.952	633 709	645.541
4.	604.694	616.342	628.063	639.857	651.725
4.1	610 756	622 441	634.198	646 029	657.934
4.2	616.843	628.564	640.358	652 226	664 167
4.3	622.954	634.712	646.543	658.447	670.425
4.4	629.089	640.884	652.752	664.693	676 707
4.5	635.249	647.081	658.985	670.963	683.014
4.6	641.434	653.302	665 243	677.257	689 345
4 7	647.643	659.547	671.525	683 576	695.701
4.8	653.876	665 817	677.832	689.919	702.081
4.9	660.134	672.112	684.163	696.287	708.485
5.	666 16	678.431	690.518	702.679	714 914

Diff.	5.0	5.1	5.2	5.3	5.4
.0	433.888	444.436	455.057	465.752	476.519
.1	439.156	449.741	460 398	471.129	481 934
.2	444 448	455 069	465.764	476 532	487.373
.3	449 765	460.423	471.154	481 958	492.836
.4	455.106	465 801	476 563	487.409	498.324
.5	460.472	471.203	482 007	492 885	503 836
.6	465.862	476.629	487.471	498.385	509 373
.7	471 276	482 081	492 958	503.909	514.934
.8	476 715	487.556	498.471	509.458	520.519
.9	482.178	493 056	504.007	515 032	526.129
1.	487.666	498 581	509 568	520.629	531.764
1.1	493.178	504.129	515 154	526 252	537.423
1.2	498.715	509.703	520 764	531.898	543.106
1 3	504 276	515.301	526.398	537.569	548.814
1.4	509 862	520 923	532 057	543.265	554 546
1.5	515.472	526.569	537 741	548.985	560.303
1.6	521.106	532.241	543.448	554 729	566 084
1.7	526.765	537 936	549.181	560.498	571.889
1.8	532.448	543 656	554 937	566.292	577.719
1.9	538.156	549.401	560.718	572 109	583.574
2.	543.888	555.169	566.524	577.952	589 453
2.1	549.645	560 963	572.354	583 818	595 356
2.2	555 426	566.781	578 208	589.709	601 284
2.3	561.232	572.623	584.087	595.625	607.236
2.4	567.062	578.489	589 991	601.565	613 213
2.5	572 916	584.381	595.918	607 529	619.214
2.6	578.795	590 296	601.871	613.518	625.239
2.7	584.698	596 236	607.847	619.532	631.289
2.8	590.626	602 201	613.848	625.569	637.364
2 9	596.578	608.189	619 874	631.632	643.463
3.	602.555	614.203	625.924	637.718	649 586
3 1	608.556	620 241	631.998	643 829	655 734
3 2	614.582	626.303	638.097	649.965	661.906
3.3	620.632	632.389	644 221	656.125	668.103
3.4	626 706	638.501	650.368	662 309	674.324
3.5	632 805	644.636	656.541	668.518	680.569
3.6	638 928	650.796	662 737	674.752	686.839
3.7	645.076	656 981	668.958	681 009	693 134
3.8	651.248	663 189	675 204	687 292	699.453
3.9	657.445	669.423	681 474	693.598	705.796
4.	663.666	675.681	687.768	699 929	712.164
4.1	669 912	681.963	694 087	706.285	718 556
4.2	676.182	688.269	700.431	712.665	724 973
4 3	682.476	694.601	706.798	719.069	731.414
4 4	688.795	700.956	713 191	725 498	737.879
4.5	695.138	707.336	719.607	731.952	744.369
4.6	701.506	713.741	726 048	738 429	750.884
4 7	707.898	720.169	732 514	744 932	757.423
4.8	714.315	726 623	739 004	751.458	763.986
4.9	720.756	733.101	745.518	758 009	770.574
5.	727.222	739.603	752.057	764.585	777.186

Diff.	5.5	5.6	5.7	5.8	5.9
.0	487.361	498.275	509.263	520.324	531.548
.1	492.812	503.763	514 787	525 885	537 056
.2	498.287	509.275	520.336	531.471	542.678
.3	503.787	514 812	525 909	537.081	548.325
.4	509.312	520.373	531.507	542.715	553.996
.5	514 861	525.958	537.129	548 374	559.692
.6	520.434	531.568	542.776	554.057	565 412
.7	526.032	537.203	548.447	559.765	571.156
.8	531 654	542.862	554.143	565.497	576.925
.9	537.301	548.545	559.863	571.254	582 718
1.	542.972	554.253	565.607	577.035	588.536
1.1	548 667	559.985	571.376	582.841	594.378
1.2	554.387	565 742	577.179	588.671	600 245
1.3	560.132	571.523	582 987	594.525	606.136
1.4	565.901	577.328	588.829	600 404	612.052
1.5	571.694	583.158	594 696	606.307	617 992
1.6	577 512	589 013	600.587	612.235	623.956
1.7	583.354	594.892	606.503	618.187	629.945
1.8	589 221	600.795	612.443	624 164	635 958
1.9	595.112	606.723	618.407	630 165	641.996
2.	601.027	612.675	624.396	636.191	648 058
2.1	606.967	618 652	630.409	642 241	654.145
2.2	612.932	624 653	636.447	648.315	660.256
2.3	618.921	630 678	642.509	654 414	666.392
2.4	624.934	636.728	648.596	660.537	672.552
2.5	630 972	642 803	654.707	666.685	678.736
2.6	637.034	648 902	660.843	672.857	684.945
2.7	643.121	655.025	667 003	679.054	691.178
2.8	649 232	661.173	673 187	685.275	697.436
2.9	655 367	667.345	679.396	691.521	703.718
3.	661.527	673 542	685.629	697.791	710.025
3.1	667.712	679.763	691 887	704 085	716 356
3 2	673 921	686.008	698.199	710.404	722.712
3.3	680.154	692.278	704.476	716.747	729.092
3.4	686 412	698 573	710.807	723 115	735.496
3.5	692.694	704.892	717.193	729.507	741.925
3.6	699.001	711.235	723 543	735.924	748.378
3.7	705.332	717.603	729 947	742.365	754.856
3 8	711 687	723.995	736 376	748 831	761.358
3.9	718.067	730.412	742.829	755.321	767.885
4.	724.472	736 853	749.307	761.835	774.436
4 1	730 901	743 318	755.809	768 374	781.012
4.2	737.354	749 808	762.336	774.937	787.612
4 3	743.832	756.323	768.887	781.525	794.236
4 4	750.334	762.862	775.463	788.137	800.885
4.5	756 861	769 425	782 063	794.774	807.558
4.6	763.412	776 013	788.687	801.435	814.256
4.7	769.987	782.625	795 336	808 121	820.978
4.8	776.587	789.262	802 009	814.831	827.725
4.9	783.212	795.923	808.707	821.565	834.496
5.	789.861	802.608	815 429	828.324	841.292

Diff.	6.0	6.1	6.2	6.3	6.4
.0	542 666	553.947	565.302	576.729	588.231
.1	548.301	559.618	571.009	582 474	594.012
.2	553.959	565.314	576.742	588.243	599.817
.3	559.643	571.034	582.498	594.036	605.647
.4	565.351	576.778	588.279	599.854	611.502
.5	571.083	582.547	594.085	605.696	617.381
.6	576.839	588.341	599.915	611.563	623.284
.7	582.621	594.158	605.769	617.454	629.212
.8	588.426	600.001	611.648	623.369	635.164
.9	594 256	605 867	617.552	629.309	641.141
1.	600.111	611.758	623.479	635.274	647.142
1.1	605.989	617.674	629.432	641.263	653.167
1.2	611.893	623.614	635.408	647.276	659.217
1.3	617.821	629.578	641.409	653.314	665.292
1.4	623.773	635.567	647.435	659.376	671 391
1.5	629.749	641.581	653.485	665 463	677.514
1.6	635.751	647.618	659.559	671.574	683.662
1.7	641.776	653.681	665.658	677.709	689.834
1.8	647.826	659.767	671.782	683.869	696.031
1.9	653.901	665.878	677.929	690.054	702.252
2.	659.999	672.014	684.102	696.263	708.497
2.1	666.123	678.174	690.298	702.496	714.767
2.2	672.271	684 358	696.519	708.754	721.062
2.3	678.443	690.567	702.765	715.036	727.381
2.4	684.639	696.801	709 035	721.343	733.724
2.5	690 861	703.058	715.329	727.674	740.092
2.6	697.106	709.341	721 648	734.029	746.484
2.7	703.376	715.647	727.992	740.409	752.901
2.8	709.671	721.978	734 359	746 814	759.342
2.9	715.989	728.334	740.752	753 243	765 807
3.	722.333	734.714	747.168	759 696	772 297
3.1	728 701	741.118	753.609	766.174	778.812
3.2	735.093	747.547	760.075	772.676	785.351
3.3	741.509	754.001	766.565	779.203	791.914
3.4	747.951	760.478	773.079	785 754	798.502
3.5	754.416	766.981	779.618	792.329	805.114
3.6	760.906	773.507	786.182	798.929	811.751
3.7	767.421	780.058	792 769	805.554	818.412
3.8	773 959	786 634	799.382	812.203	825 097
3.9	780.523	793.234	806 018	818 876	831.807
4.	787.111	799.858	812.679	825 574	838 542
4.1	793.723	806.507	819.365	832.296	845.301
4.2	800.359	813.181	826.075	839.043	852 084
4.3	807.021	819.878	832 809	845 814	858.892
4.4	813.706	826.601	839.568	852.609	865.724
4 5	820.416	833.347	846 352	859.429	872.581
4.6	827.151	840 118	853.159	866.274	879.462
4.7	833.909	846.914	859.992	873.143	886.367
4.8	840.693	853.734	866.848	880.036	893.297
4.9	847.501	860.578	873.729	886.954	900.252
5.	854.333	867.447	880.635	893.896	907 231

Diff.	6.5	6.6	6.7	6.8	6.9
.0	599 805	611.453	623.174	634.968	646 836
.1	605.623	617 307	629.065	640 896	652.801
.2	611 465	623.186	634.981	646.848	658 789
.3	617.332	629.089	640.921	652 825	664.803
.4	623 223	635.017	646 885	658 826	670.841
.5	629.138	640.969	652 874	664 852	676 903
.6	635.078	646.946	658 887	670.902	682.989
.7	641.043	652.947	664.925	676.976	689.101
.8	647.032	658.973	670.987	683.075	695.236
.9	653.045	665 023	677.074	689.198	701.396
1.	659 083	671.097	683.185	695.346	707 531
1.1	665.145	677.196	689.324	701.518	713.789
1.2	671.232	683.319	694.481	707.715	720.023
1.3	677.343	689.467	701.665	713 936	726 281
1.4	683.478	695 639	707 874	720.182	732 563
1.5	689.638	701.836	714.107	726 452	738.869
1.6	695 823	708.057	720 365	732.746	745 201
1.7	702.032	714 303	726.647	739 065	751 556
1.8	708 265	720.573	732.954	745 408	757 936
1.9	714.523	726 867	739.285	751 776	764.341
2.	720 805	733.186	745 641	758.168	770 769
2.1	727.112	739.529	752 021	764.585	777 223
2.2	733 443	745 897	758 425	771.026	783.701
2.3	739.798	752 289	764 854	777.492	790 203
2.4	746.178	758.706	771 307	783 982	796.729
2.5	752.583	765.147	777.785	790 496	803.281
2.6	759.012	771.613	784 287	797 035	809.856
2.7	765.465	778.103	790.814	803.598	816 456
2.8	771.943	784 617	797 365	810.186	823.081
2.9	778 445	791.156	803.941	816 798	829.729
3.	784.972	797.719	810 541	823.435	836 403
3.1	791.523	804.307	817.165	830 096	843.101
3.2	798 098	810.919	823.814	836.782	849.823
3.3	804.698	817.556	830.487	843.492	856.569
3.4	811.223	824.217	837 185	850 226	863.341
3.5	817.972	830.903	843.907	856.985	870.136
3.6	824 645	837 613	850.654	863.768	876.956
3.7	831 343	844.347	857.426	870.576	883 801
3.8	838.065	851 106	864.221	877.408	890.669
3.9	844.812	857.889	871.041	884.265	897 563
4.	851.583	864.697	877 885	891.146	904.481
4.1	858 378	871.529	884.754	898.052	911.423
4.2	865.198	878.386	891.647	904.982	918.389
4.3	872.043	885.267	898 565	911.936	925.381
4.4	878 912	892.173	905 507	918.915	932.396
4.5	885.805	899 103	912.474	925 918	939.436
4.6	892.723	906 057	919.465	932 946	946 501
4.7	899.665	913 036	926.481	939.998	953.589
4.8	906 632	920 039	933.521	947.076	960.703
4.9	913.623	927 067	940.585	954.176	967.841
5.	920 638	934.119	947 674	961.362	975.003

Diff.	7.0	7.1	7.2	7.3	7.4
.0	658.777	670.792	682.879	695.041	707.275
.1	664.778	676.829	688 954	701.152	713.423
.2	670.804	682.892	695.053	707.287	719 595
.3	676 854	688.978	701.176	713.447	725.792
.4	682.928	695.089	707.324	719 632	732.013
.5	689.027	701.225	713.496	725.841	738.258
.6	695.151	707.385	719.693	732 074	744.528
.7	701.298	713.569	725.914	738.332	750.823
.8	707.471	719.778	732.159	744.614	757.142
.9	713.667	726.012	738.429	750.921	763 485
1.	719.888	732.269	744.724	757.252	769.853
1.1	726.134	738.552	751.043	763.607	776.245
1.2	732.404	744 858	757.386	769 987	782 662
1.3	738.698	751.189	763.754	776.392	789.103
1.4	745.017	757.545	770 146	782.821	795.568
1.5	751.361	763.925	776.563	789.274	802.058
1.6	757.728	770.329	783.004	795.752	808.573
1.7	764.121	776.758	789 469	802.254	815.112
1.8	770.537	783.212	795.959	808.781	821.675
1.9	776.978	789.689	802.474	815.332	828.263
2.	783.444	796.192	809.013	821.907	834.875
2.1	789.934	802.718	815.576	828.507	841.512
2.2	796.448	809 269	822 164	835 132	848.173
2.3	802.987	815 845	828 776	841.741	854.858
2.4	809.551	822.445	835 413	848 454	861.568
2.5	816.138	829.069	842.074	855.152	868.303
2.6	822 751	835.718	848.759	861.874	875.062
2 7	829.387	842.392	855.469	868.621	881 845
2 8	836.048	849.089	862 204	875.392	886 653
2.9	842.734	855 812	868.963	882.187	895 485
3.	849.444	862.558	875.746	889.007	902.342
3.1	856.178	869 329	882.554	895.852	909.223
3.2	862.937	876.125	889.386	902.721	916.128
3.3	869 721	882.945	896 243	909 614	923.058
3.4	876 528	889 789	903.124	916 532	930.013
3.5	883.361	896.658	910.029	923.474	936 992
3.6	890.217	903.552	916.959	930.441	943 995
3.7	897.098	910.469	923.914	937.432	951.023
3.8	904 004	917.412	930.893	944.447	958.075
3.9	910.934	924.378	937.896	951.487	965.152
4.	917.888	931 369	944.924	958.552	972.253
4.1	924.867	938.385	951.976	965.641	979 378
4.2	931.871	945.425	959.053	972.754	986.528
4.3	938.898	952.489	966.154	979.892	993.703
4.4	945 951	959 578	973.279	987.054	1000 902
4.5	953.027	966 692	980.429	994 241	1008.125
4.6	960.128	973.829	987.604	1001.452	1015.373
4.7	967.254	980.992	994.803	1008.687	1022.645
4.8	974.404	988 178	1002 026	1015.947	1029.942
4.9	981.578	995.389	1009.274	1023.232	1037.263
5.	988.777	1002.625	1016 546	1030 541	1044 608

Diff.	7.5	7.6	7.7	7.8	7.9
.0	719.583	731.964	744.418	756.946	769.547
.1	725.767	738.185	750.676	763.241	775.878
.2	731.976	744.431	756.958	769.559	782.234
.3	738.209	750.701	763.265	775.903	788.614
.4	744 467	756.995	769.596	782.271	795.018
.5	750 749	763.314	775 952	788 663	801.447
.6	757.056	769.657	782.332	795.079	807.901
.7	763.387	776.025	788.736	801.521	814.378
.8	769.743	782.417	795.165	807.986	820.881
.9	776.123	788.834	801.618	814.476	827.407
1.	782.527	795.275	808.096	820.991	833.958
1.1	788.956	801.741	814.598	827 529	840.534
1.2	795.409	808.231	821.125	834.093	847 134
1.3	801.887	814.745	827.676	840.681	853.758
1.4	808.389	821.284	834.252	847.293	860.407
1.5	814.916	827.847	840 852	853.929	867.081
1.6	821.467	834.435	847.476	860 591	873.778
1.7	828.043	841 047	854.125	867.276	880.501
1.8	834.643	847.684	860.798	873.986	887.247
1.9	841.267	854.345	867.496	880.721	894.018
2.	847.916	861.031	874.218	887.479	900.814
2.1	854.589	867.741	880.965	894.263	907.634
2.2	861.287	874.475	887.736	901.071	914 478
2.3	868.009	881.234	894 532	907.903	921.347
2.4	874.756	888 017	901 352	914.759	928.241
2.5	881.527	894.825	908.196	921.641	935.158
2.6	888.323	901.657	915 065	928.546	942.101
2.7	895.143	908.514	921.958	935.476	949 067
2.8	901.987	915.395	928 876	942.431	956 058
2.9	908.856	922.301	935.818	949.409	963.074
3.	915.749	929.231	942.785	956.413	970.114
3.1	922.667	936.185	949.776	963.441	977.178
3.2	929.609	943.164	956.792	970 493	984.267
3 3	936.576	950.167	963.832	977.569	991 381
3.4	943.567	957.195	970.896	984.671	998.518
3.5	950.583	964.247	977.985	991.796	1005.681
3.6	957.623	971 324	985.098	998.946	1012 867
3.7	964 687	978.425	992.236	1006.121	1020.078
3.8	971.776	985.551	999.398	1013.319	1027.314
3.9	978.889	992 701	1006 585	1020.543	1034.574
4.	986.027	999 875	1013.796	1027.791	1041.858
4.1	993 189	1007.074	1021.032	1035.063	1049.167
4.2	1000 376	1014.297	1028 292	1042.359	1056 501
4.3	1007.587	1021.545	1035.576	1049.681	1063 858
4.4	1014.823	1028.817	1042.885	1057.026	1071.241
4 5	1022 083	1036 114	1050.218	1064.396	1078.647
4.6	1029.367	1043.435	1057 576	1071.791	1086.078
4.7	1036 676	1050.781	1064.958	1079 209	1093.534
4.8	1044.009	1058 151	1072.365	1086.653	1101.014
4.9	1051 367	1065.545	1079.796	1094.121	1108.518
5.	1058 749	1072.964	1087.252	1101.613	1116.047

Diff.	8.0	8.1	8.2	8.3	8.4
.0	782.222	794.969	807.791	820.685	833.653
.1	788.589	801.374	814.232	827.163	840.167
.2	794.982	807.803	820.697	833.665	846.706
.3	801.398	814.256	827.187	840.192	853.269
.4	807.839	820.734	833.702	846.743	859.857
.5	814.305	827.236	840.241	853.318	866.469
.6	820.795	833.763	846.804	859.918	873.106
.7	827.309	840.314	853.392	866.543	879.767
.8	833.848	846.889	860.004	873.192	886.453
.9	840.412	853.489	866.641	879.865	893.163
1.	846.999	860.114	873.302	886 563	899.897
1.1	853.612	866.763	879.987	893 285	906.656
1.2	860.248	873.436	886.697	900.032	913.439
1.3	866.909	880.134	893.432	906.803	920.247
1.4	873.595	886.856	900.191	913.598	927.079
1.5	880.305	893.603	906.974	920.418	933.936
1.6	887 039	900.374	913.782	927.263	940.817
1.7	893.798	907.169	920.614	934.132	947.723
1.8	900.582	913.989	927.471	941 025	954.653
1.9	907.389	920 834	934.352	947.943	961.607
2.	914.222	927.703	941.257	954.885	968.586
2.1	921.078	934 596	948.187	961.852	975.589
2.2	927.959	941.514	955.142	968.843	982 617
2 3	934.865	948.456	962.121	975.858	989.669
2.4	941.795	955.423	969.124	982.898	996.746
2.5	948.749	962 414	976.152	989.963	1003.847
2.6	955.728	969.429	983.204	997.052	1010.973
2.7	962.732	976.469	990.281	1004.165	1018.123
2.8	969.759	983.534	997.382	1011.303	1025 297
2.9	976.812	990.663	1004 507	1018.465	1032.496
3.	983.888	997.736	1011.657	1025.652	1039.719
3.1	990.989	1004.874	1018.832	1052.863	1046.967
3.2	998.115	1012 036	1026.031	1040.098	1054.239
3.3	1005.265	1019 223	1033 254	1047.358	1061.536
3.4	1012.439	1026.434	1040.502	1054.643	1068.857
3.5	1019.638	1033.669	1047 774	1061.952	1076.203
3.6	1026.862	1040.929	1055 071	1069.285	1083.573
3.7	1034.109	1048.214	1062.392	1076 643	1090.967
3 8	1041.382	1055.523	1069.737	1084.025	1098.386
3.9	1048.678	1062.856	1077.107	1091.432	1105.829
4.	1055.999	1070.214	1084.502	1098.863	1113.297
4.1	1063.345	1077 596	1091.921	1106.318	1120.789
4.2	1070.715	1085.003	1099.364	1113.798	1128.306
4.3	1078.109	1092.434	1106.832	1121.303	1135.847
4.4	1085.528	1099.889	1114.324	1128.832	1143.413
4.5	1092.972	1107.369	1121.841	1136.385	1151.003
4.6	1100.439	1114.874	1129.382	1143.963	1158.617
4.7	1107.932	1122.403	1136.947	1151.565	1166.256
4.8	1115.448	1129.956	1144.537	1159.192	1173.919
4 9	1122.989	1137.534	1152.152	1166.843	1181.607
5.	1130.555	1145.136	1159.791	1174.518	1189.319

Diff.	8.5	8.6	8.7	8.8	8.9
.0	846 694	859.808	872.996	886.257	899.592
.1	853.245	866 396	879.621	892.918	906.289
.2	859.821	873 008	886.269	899.604	913.012
.3	866.421	879.645	892.943	906.314	919.758
.4	873.045	886 306	899.641	913.048	926.529
.5	879.694	892.992	906.363	919 807	933.325
.6	886.367	899.702	913.109	926.591	940.145
.7	893.065	906 436	919.881	933.398	946.989
.8	899.787	913 195	926 676	940.231	953.858
.9	906.534	919.978	933.496	947.087	960.752
1.	913.305	926.786	940.341	953.968	967.669
1.1	920.101	933.618	947.209	960.874	974.612
1 2	926.921	940.475	954.103	967.804	981.578
1.3	933.765	947.356	961.021	974.758	988.569
1.4	940.634	954 262	967.963	971.737	995.585
1 5	947.527	961.192	974.929	988.741	1002.625
1.6	954 445	968.146	981.921	995.768	1009 689
1.7	961.387	975.125	989 936	1002.821	1016.778
1.8	968 354	982.128	995.976	1009.897	1023.892
1.9	975.345	989.156	1003.041	1016.998	1031.029
2.	982.361	996.208	1010.129	1024.124	1038.192
2.1	989.401	1003.285	1017.243	1031.274	1045.378
2.2	996 465	1010.386	1024 381	1038 448	1052.589
2.3	1003.554	1017 512	1031.543	1045.647	1059.825
2.4	1010 667	1024.662	1038 729	1052.871	1067.085
2.5	1017.805	1031 836	1045.941	1060.118	1074.369
2.6	1024.967	1039 035	1053.176	1067.391	1081.678
2.7	1032 154	1046.258	1060.436	1074.687	1089.012
2 8	1039.365	1053 506	1067.721	1082.008	1096.369
2 9	1046.601	1060.778	1075 029	1089.354	1103.752
3.	1053 861	1068.075	1082.363	1096.724	1111.158
3.1	1061.145	1075.396	1089 721	1104.118	1118.589
3.2	1068.454	1082.742	1097.103	1111.537	1126.045
3.3	1075.787	1090.112	1104.509	1118.981	1133 525
3 4	1083.145	1097.506	1111.941	1126.448	1141.029
3.5	1090.527	1104 925	1119.396	1133.941	1148.558
3 6	1097.934	1112.368	1126.876	1141.547	1156.112
3.7	1105 365	1119 836	1134 381	1148.998	1163.689
3.8	1112 821	1127.328	1141.909	1156 564	1171.292
3.9	1120 301	1134.845	1149.463	1164.154	1178 918
4.	1127.805	1142.386	1157.041	1171.768	1186.569
4 1	1135.334	1149 952	1164.643	1179.407	1194.245
4.2	1142 887	1157 543	1172.269	1187.071	1201.945
4 3	1150 465	1165.156	1179.921	1194.758	1209.669
4 4	1158.067	1172.795	1187.596	1202 471	1217.418
4.5	1165.694	1180.458	1195.296	1210 207	1225.192
4 6	1173.345	1188 146	1203 021	1217.968	1232.989
4 7	1181.021	1195.858	1210 769	1225.754	1240.812
4.8	1188 721	1203.595	1218 543	1233.564	1248.658
4.9	1196.445	1211.356	1226.341	1241.398	1256.529
5.	1204 194	1219.142	1234.163	1249.257	1264.425

Diff.	9.0	9.1	9.2	9.3	9.4
.0	912.999	926.481	940.035	953.663	967.364
.1	919.734	933.252	946.843	960.507	974.245
.2	926.493	940.047	953.675	967.376	981.151
.3	933.276	946.867	960.532	974.269	988.081
.4	940.084	953.712	967.413	981.187	995.035
.5	946.916	960.581	974.318	988.129	1002.014
.6	953 773	967.474	981.248	995.096	1009.017
.7	960.654	974.392	988.203	1002.087	1016.045
.8	967.559	981.334	995.182	1009.103	1023.097
.9	974 489	988.301	1002.185	1016.143	1030.174
1.	981.444	995.292	1009.213	1023.207	1037.275
1.1	988.423	1002.307	1016.265	1030.296	1044.401
1 2	995.426	1009.347	1023.342	1037.409	1051.551
1.3	1002.454	1016.412	1030.443	1044.547	1058.725
1.4	1009 506	1023.501	1037.568	1051.709	1065.924
1.5	1016.583	1030.614	1044 718	1058.896	1073.147
1.6	1023.684	1037.752	1051 893	1066.107	1080.395
1.7	1030.809	1044.914	1059.092	1073.343	1087.667
1.8	1037.959	1052.101	1066.315	1080.603	1094.964
1.9	1045.134	1059.312	1073.563	1087.887	1102.285
2.	1052 333	1066.547	1080.835	1095.196	1109 631
2.1	1059.556	1073.807	1088.132	1102.529	1117.001
2.2	1066.804	1081 092	1095.453	1109 887	1124.395
2.3	1074 076	1088 401	1102 798	1117.269	1131.814
2.4	1081.373	1095.734	1110.168	1124.676	1139.257
2 5	1088 694	1103.092	1117.563	1132.107	1146.725
2.6	1096.039	1110.474	1124 982	1139.563	1154.217
2.7	1103.409	1117.881	1132.425	1147.043	1161.734
2.8	1110 804	1125.312	1139.893	1154.547	1169.275
2.9	1118.223	1132.767	1147.385	1162.076	1176.841
3.	1125 666	1140.247	1154.902	1169.629	1184.431
3.1	1133.134	1147.752	1162.443	1177.207	1192.045
3.2	1140 626	1155.281	1170.008	1184.809	1199.684
3.3	1148.143	1162.834	1177.598	1192 436	1207.347
3.4	1155.685	1170.412	1185.213	1200.087	1215.035
3.5	1163.249	1178.014	1192.852	1207 763	1222.747
3.6	1170 839	1185 641	1200 515	1215.463	1230.484
3.7	1178.454	1193.292	1208.203	1223.187	1238.245
3.8	1186.093	1200.967	1215.915	1230.936	1246.031
3.9	1193.756	1208.667	1223.652	1238.709	1253 841
4.	1201 444	1216.392	1231.413	1246.507	1261.675
4.1	1209.156	1224.141	1239.198	1254.329	1269.534
4.2	1216.893	1231.914	1247.008	1262.176	1277.417
4.3	1224.654	1239.712	1254.843	1270.047	1285.325
4.4	1232.439	1247.534	1262.702	1277 943	1293.257
4.5	1240.249	1255.381	1270.585	1285 863	1301.214
4.6	1248.084	1263.252	1278.493	1293.807	1309.195
4 7	1255.943	1271.147	1284.425	1301.776	1317.201
4.8	1263.826	1279.067	1294 382	1309.769	1325.231
4.9	1271.734	1287.012	1302.363	1317.787	1333.285
5.	1279 666	1294.981	1310.368	1325 829	1341.364

Diff.	9.5	9.6	9.7	9.8	9.9
.0	981.138	994.986	1008.907	1022.902	1036 969
.1	988.056	1001.941	1015.898	1029.929	1044.034
.2	994.998	1008 919	1022.914	1036.982	1051.123
.3	1001.965	1015.923	1029.954	1044.058	1058.236
.4	1008.956	1022.951	1037.018	1051.159	1065.374
.5	1015.972	1030.003	1044.107	1058.285	1072.536
.6	1023 012	1037.079	1051.221	1065.435	1079.723
.7	1030.076	1044.181	1058.358	1072.609	1086.934
.8	1037.165	1051.307	1065.521	1079 808	1094.169
.9	1044.278	1058.456	1072.707	1087.032	1101.429
1.	1051.416	1065.631	1079.918	1094.279	1108 714
1.1	1058.578	1072 829	1087.154	1101.552	1116.023
1.2	1065 765	1080.053	1094 414	1108.848	1123.356
1 3	1072.976	1087.301	1101.698	1116.169	1130.714
1.4	1080.212	1094.573	1109 007	1123.515	1138.096
1.5	1087.472	1101.869	1116 341	1130.885	1145.503
1.6	1094.756	1109 191	1123.698	1138.279	1152.934
1.7	1102.065	1116.536	1131.081	1145.698	1160.389
1.8	1109.398	1123 906	1138.487	1153.142	1167 869
1.9	1116.756	1131.301	1145 918	1160 609	1175.374
2.	1124.138	1138.719	1153.374	1168.102	1182.903
2.1	1131.545	1146.163	1160.854	1175 618	1190 456
2.2	1138.976	1153.631	1168.358	1183.159	1198.034
2.3	1146.432	1161.123	1175.887	1190.725	1205 636
2.4	1153.912	1168.639	1183.441	1198.315	1213.263
2.5	1161.416	1176.181	1191.018	1205.929	1220.914
2 6	1168.945	1183.746	1198 621	1213.568	1228.589
2.7	1176.498	1191.336	1206 247	1221 232	1236 289
2.8	1184 076	1198.951	1213 898	1228.919	1244.014
2 9	1191.678	1206.589	1221.574	1236.632	1251.763
3.	1199.305	1214 253	1229.274	1244.368	1259 536
3 1	1206.956	1221 941	1236.998	1252.129	1267.334
3 2	1214 632	1229.653	1244.747	1259.915	1275.156
3.3	1222.332	1237.389	1252.521	1267 725	1283.003
3.4	1230.056	1245.151	1260.318	1275.559	1290.874
3.5	1237.805	1252.936	1268.141	1283.418	1298.769
3.6	1245.578	1260.746	1275.987	1291.302	1306.689
3.7	1253.376	1268.581	1283 858	1299.209	1314.634
3.8	1261.198	1276.439	1291.754	1307.142	1322.603
3.9	1269.045	1284.323	1299.674	1315.098	1330.596
4.	1276.916	1292.231	1307.618	1323 079	1338.614
4 1	1284.812	1300.163	1315.587	1331 085	1346.656
4.2	1292.732	1308.119	1323.581	1339.115	1354.723
4 3	1300.676	1316 101	1331.598	1347.169	1362.814
4 4	1308.645	1324.106	1339.641	1355.248	1370.929
4.5	1316.638	1332.136	1347.707	1363.352	1379.069
4.6	1324.656	1340.191	1355.798	1371.479	1387.234
4 7	1332.698	1348.269	1363.914	1379.632	1395.423
4.8	1340 765	1356.373	1372.054	1387.808	1403.636
4.9	1348 856	1364 501	1380 218	1396.009	1411.874
5.	1356.972	1372.653	1388 407	1404.235	1420.136

NOTES.

Table 5 is predicated on the supposition, that the surface of the ground contained in the *bed* of the canal, is *horizontal*, or *inclined* in the direction of the *route* of the canal, agreeably to Case 2 of Problem ix. It is calculated for every *tenth of a foot* variation of slope from *level ground*, to ground sloping *five feet* in the distance of *one chain*, and for every *tenth of a foot cutting* from bottom (0) to 10 feet. If it is desired to extend this table either for slope or cutting, it will be observed that the second difference of any two contiguous numbers denoting cubic yards, is constant and equal; this difference, therefore, is denominated the common difference, and the extension may be made by adding continually this common difference to the numbers and differences preceding.

For the most ordinary cases of canal cutting, occurring on gentle side-long ground, this table may be used without much error. When the transverse inclination, however, is considerable, the *prism* considered in Case 3 of Prob. ix. should be added. A table showing the value of this prism for different lateral slopes would have been given, but the limits prescribed for this work precluded it; its place may be easily supplied from the rules already laid down.

As Table 5 is intended to meet those cases most common in canal cutting, and is probably the only one of the kind ever published, at least in this country, it is believed, that when properly understood, it will be found a very useful auxiliary to engineers, contractors, and all other persons employed on, or interested in the excavation of the Pennsylvania Canal, or other canals of a like dimension.

Under the description and use of this table, it has been shown how the determination of the quantity of *canal* or *tow-path* embankment may be facilitated: by a similar method also, may the excavation be found when the size of the intended canal is either greater or less than that for which the table has been particularly arranged; that part of the mensuration which is generally considered the most difficult being exhibited by the table.

Although the question to find the solidity of a piece of earth to be excavated from the *trunk* of a canal, when the cuttings at the ends of the piece are unequal, is a plain and simple one; yet it is known that several erroneous methods are sometimes adopted for its solution. For instance; let the cutting in one chain length of canal, 28 feet wide at bottom, with sides sloping as $1\frac{1}{2}$ to 1. vary from 5 to 10 feet. To find the contents of the piece in cubic yards.

First—By taking the half sum of the two depths, or cutting, for the depth of the mean area, as it is called, and from thence finding the area of the cross section at this depth, multiplying it by $22 \div 9$, we shall have 719.583 cubic yards.

Secondly—Multiplying half the sum of the *areas* of the two ends by $22 \div 9$, will give the product equal $742\frac{1}{2}$ cubic yards.

The correct measurement, as will be made by the rule given in Case 2 of Prob. ix. or by a reference to Table 5, is $727\frac{2}{9}$ cubic yards. The first method therefore gives 7 cubic yards of earth too little, and the latter 15 cubic yards too much. The inaccuracies of the above methods might have been shown mathematically, but this comparison it is thought will be sufficiently satisfactory, and more likely to be generally understood.

CONCLUSION.

The rules and information contained in the preceding pages, are respectfully submitted to the consideration and use of all persons employed as engineers, contractors, &c. in the construction of the Pennsylvania Canal, as well as to those in other states, where similar internal improvements are in progression.

A principal object of this publication has been to form for the use of young men, who have been introduced into the service of the state, as students to the principles and practice of canal engineering, a code of the most useful and important problems relating to their duties, with solutions as concise as the nature of the subject, and a due regard to perspicuity would admit, so as to form a text or reference book convenient for the field, and at small expense.

Several miscellaneous articles are added on subjects of frequent occurrence, a knowledge of which is considered highly important to a canal engineer.

It cannot fail to appear, from even the few preceding inquiries, which are perhaps among the simplest that occur in the profession, how much the advancement of an engineer depends, or ought to depend, upon a correct and competent knowledge of mathematics. It has been observed, "that none ought to be admitted to that honourable distinction, who are unlearned therein, however much they may have seen, or even executed, under the orders of abler men." In almost all the various departments of engineering, *it* is the *touch-stone* of perfection. To the student who desires to become eminent in his profession, it is therefore recommended to let slip no opportunity of cultivating a knowledge of this important science.

THE END.

Fig. 1.

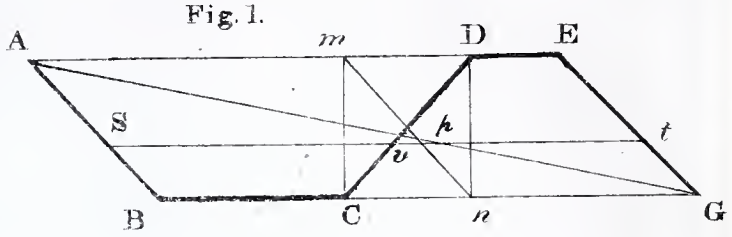
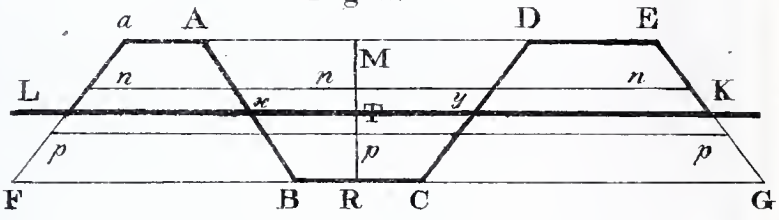


Fig. 2.



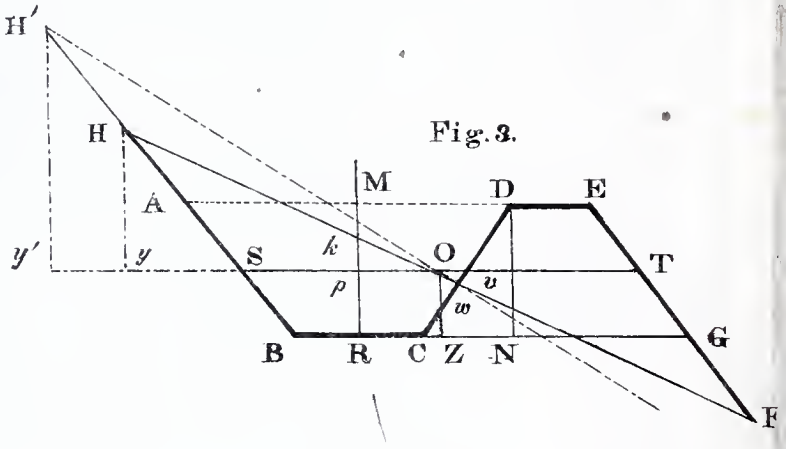
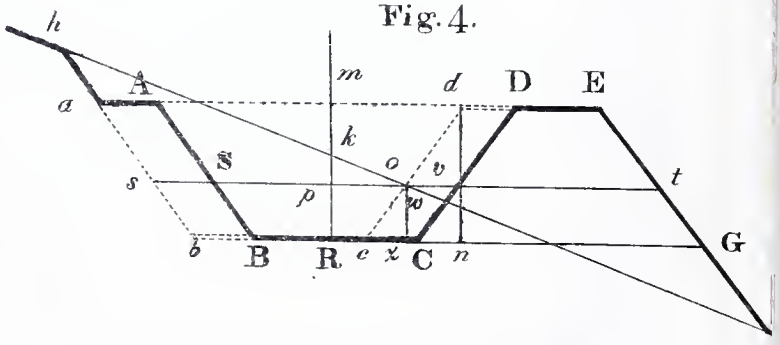


Fig. 4.



Lucas' Lithography.

Plate II.

Fig. 5.

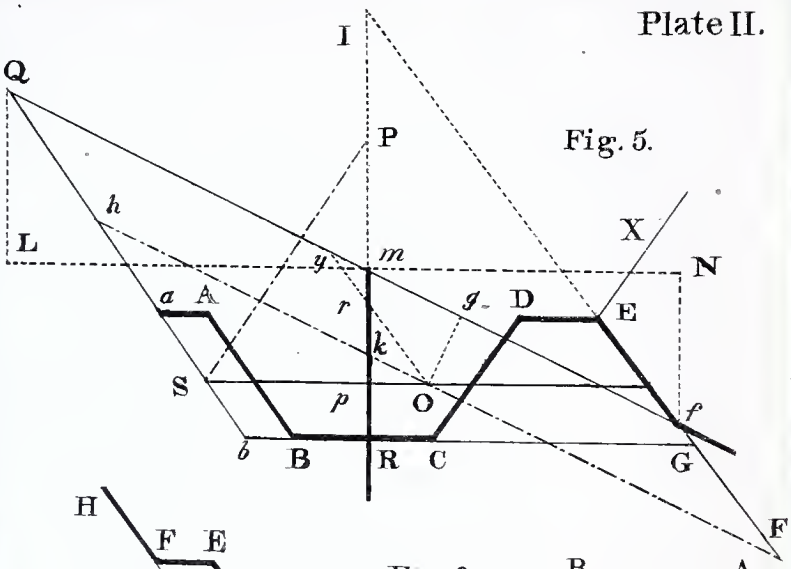


Fig. 6.

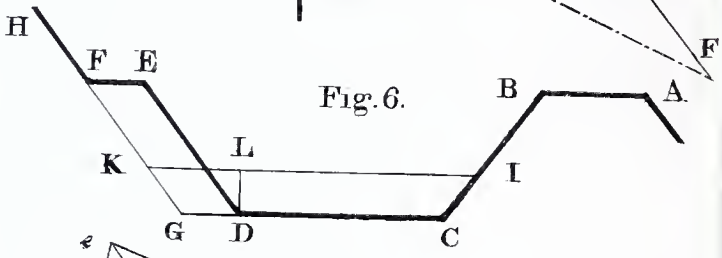


Fig. 7.

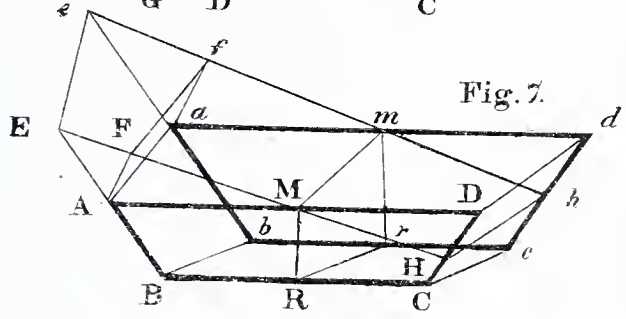
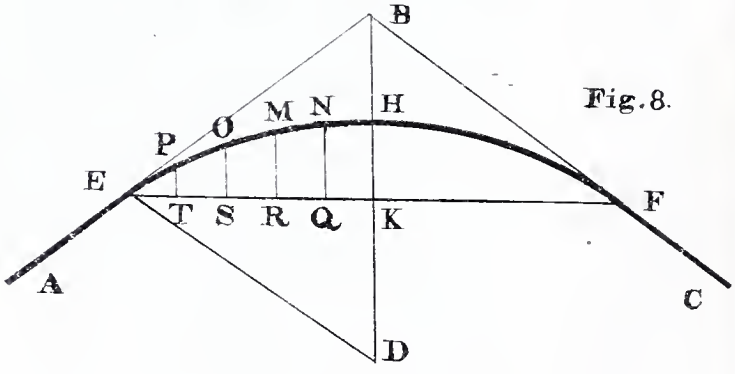


Fig. 8.



Lucas' Lithography